11   Quantum Computing (AD)

(*a*) (*i*)   Write out the matrix for the three-qubit Toffoli gate.   [2 marks]

(*ii*) A three-qubit system is in the state

$$\frac{1}{2}(|000\rangle + |101\rangle + |110\rangle + |111\rangle).$$

The Toffoli gate is applied to this system and then a measurement is performed on the first qubit. What is the probability of observing a $|1\rangle$? Show all working out.   [2 marks]

(*b*) State and prove the *no-cloning theorem*.   [6 marks]

(*c*) Suppose we are given a qubit $|\theta\rangle$ in an unknown state, but we know it is either $|0\rangle$ or $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. We would like to devise a circuit to determine, with certainty, which of the two is the case, using any number of unitary and measurement operations.

Either explain how to construct such a circuit, or prove that no such circuit is possible.   [10 marks]