1 Advanced Graphics (NAD)

(a) In at most five sentences, describe the *kd-tree* data structure and its use in ray tracing.   [3 marks]

(b) Several formulae were discussed in the course for calculating the *normal to a discrete surface* at a vertex.

   (i) In at most five sentences, explain why all of these formulae are necessarily approximations; and why a true answer is impossible.   [3 marks]

   (ii) Give the best (most nearly accurate) formula discussed in the course.
                                                                                                              [1 mark]

(c) The *angle deficit* is a method for measuring discrete curvature. Give the formula for the *angle deficit* of a vertex v.   [1 mark]

(d) A simple closed discrete surface has 148 vertices, 248 edges and 80 faces. What must its genus be, and why?   [2 marks]

(e) An implicit surface system is described by the force function $f(r) = 1/r^2$, the threshold $t = 1.0$, and a set of generating points.

   (i) A surface is defined by two generating points at $(-x, 0, 0)$ and $(x, 0, 0)$. What is the largest value of x such that the surface forms a single component?   [2 marks]

   (ii) A surface is defined by three generating points, positioned in an equilateral triangle at $(-1, 0, 0)$, $(1, 0, 0)$, and $(0, \sqrt{3}, 0)$. Does the surface form a single component of genus 0, a single component of genus 1, or three separated components of genus 0?   [1 mark]

   (iii) If the set of generating points is described as $v \in \mathbf{V}$ we can write the equation for the total value of the force $F(x)$ at a point x in space as

$$F(x) = \sum_{v \in \mathbf{V}} \frac{1}{|x - v|^2}$$

   Give an expression for $F(x)$ if the surface were instead defined by infinite *generating lines* of force, where each generating line, L, is expressed by two points $[A, B]$. If your expression is correct, the implicit surface described by a single line $[A, B]$ will be an infinite cylinder of radius one centred on the axis $[A, B]$.   [3 marks]

   (iv) Modify your definition to treat each generating line as a finite line segment from A to B. A single generating line should describe a surface that is a finite cylinder with spherical end caps.   [2 marks]

   (v) One alternative force function, for use with generating points, is:

$$f(r) = \begin{cases} 1 - \frac{4}{9}\frac{r^6}{R^6} + \frac{17}{9}\frac{r^4}{R^4} - \frac{22}{9}\frac{r^2}{R^2} & r < R \\ 0 & r \geq R \end{cases}$$

   Suggest why and when this function is more computationally efficient than the function $f(r) = 1/r^2$.   [2 marks]