3 Discrete Mathematics I (SS)

(a) Which of the following formulas are tautologies? Explain what is meant by "tautology" and write down truth tables to justify your answers.

(i) $p \Rightarrow q$

(ii) $(p \Rightarrow q) \Rightarrow p$

(iii) $((p \Rightarrow q) \Rightarrow p) \Rightarrow p$ [4 marks]

(b) Recall the following introduction and elimination rules for implication.

$\ldots$

| $m.$ Assume P |
| $\ldots$ |
| $n.$ Q from $\ldots$ by $\ldots$ |

$n + 1.$ $P \Rightarrow Q$ from m–n,
    by $\Rightarrow$-introduction.

$\ldots$

$l.$ $P \Rightarrow Q$ from $\ldots$ by $\ldots$

$\ldots$

$m.$ P from $\ldots$ by $\ldots$

$\ldots$

$n.$ Q from l, m
    by $\Rightarrow$-elimination.

(i) Write down the elimination rules for negation and falsity. [3 marks]

(ii) Using the four rules above, write down a structured proof of

$$\neg p \Rightarrow (p \Rightarrow q)$$

[4 marks]

(iii) Write down the principle of proof by contradiction. [2 marks]

(iv) Using everything from part (b) so far, write down a structured proof of

$$((p \Rightarrow q) \Rightarrow p) \Rightarrow p$$

[7 marks]