2011 Paper 6 Question 9

Semantics of Programming Languages

The following grammar specifies the syntax of a simple imperative programming language. It is a fragment of L3.

Values:	v	::=	**skip** $\mid n \mid x \mid \ell$
			(n ranges over integers, x over variables, and ℓ over locations)
Expressions:	e	::=	$v \mid \textbf{let } x = e \textbf{ in } e' \mid v + v' \mid v := v' \mid\, !v \mid \textbf{ref}(v)$
Types:	T	::=	$\textbf{unit} \mid \textbf{int} \mid T\,\textbf{ref}$
Stores:	s		finite partial functions from locations to values
Environments:	Γ		finite partial functions from locations and variables to types

Note that the grammar is very restrictive. For instance, the expression $(3+4)+7$ is not allowed.

The language is typed according to the following standard rules.

$$\frac{}{\Gamma \vdash \textbf{skip} : \textbf{unit}} \qquad \frac{}{\Gamma \vdash n : \textbf{int}} \text{ for } n \text{ an integer}$$

$$\frac{}{\Gamma \vdash x : T} \text{ if } \Gamma(x) = T \qquad \frac{}{\Gamma \vdash \ell : T\,\textbf{ref}} \text{ if } \Gamma(\ell) = T\,\textbf{ref}$$

$$\frac{\Gamma \vdash e : T \quad \Gamma, x{:}T \vdash e' : T'}{\Gamma \vdash \textbf{let } x = e \textbf{ in } e' : T'} \qquad \frac{\Gamma \vdash v : \textbf{int} \quad \Gamma \vdash v' : \textbf{int}}{\Gamma \vdash v + v' : \textbf{int}}$$

$$\frac{\Gamma \vdash v : T\,\textbf{ref} \quad \Gamma \vdash v' : T}{\Gamma \vdash v := v' : \textbf{unit}} \qquad \frac{\Gamma \vdash v : T\,\textbf{ref}}{\Gamma \vdash\, !v : T} \qquad \frac{\Gamma \vdash v : T}{\Gamma \vdash \textbf{ref}(v) : T\,\textbf{ref}}$$

(*a*) Give a reasonable operational semantics for this language by defining a relation over configurations. [7 marks]

(*b*) Write down all the reduction steps of the following expression. You do not need to give their derivations.

$$\textbf{let } x = \textbf{ref}(0) \textbf{ in let } y = !x \textbf{ in let } z = y + 3 \textbf{ in } x := z$$

[3 marks]

(*c*) State and prove a Type Preservation Theorem for this language.

You may assume the following definition:

a store s is *well-typed for* Γ, written $\Gamma \vdash s$,
if for all locations $\ell \in \text{dom}(s)$, there is a type T
such that $\Gamma(\ell) = T\,\textbf{ref}$ and $\Gamma \vdash s(\ell) : T$

You may also assume the following substitution lemma:

$$\text{If } \Gamma \vdash v : T \text{ and } \Gamma, x : T \vdash e : T' \text{ with } x \notin \text{dom}(\Gamma) \text{ then } \Gamma \vdash \{v/x\}e : T'$$

[10 marks]