2010 Paper 1 Question 4

Discrete Mathematics I

Let x, y, z range over individuals I and a, b range over societies S. Let M, F and T be atomic predicates as follows:

$$
\begin{array}{ll}
M(x, a) & x \text{ is a member of society } a \\
F(a) & \text{society } a \text{ involves fighting} \\
T(x, y, a) & x \text{ talks to } y \text{ about } a
\end{array}
$$

(a) Formalise each of the following English statements and translate each of the following formulae into idiomatic English (natural English sentences).

(i) $\forall x, y, a.\, T(x, y, a) \Rightarrow T(y, x, a)$

(ii) Nobody talks to themselves about anything.

(iii) There's at most one society involving fighting.

(iv) All societies have at least two members.

(v) $\forall a.(\exists x, y.(M(x, a) \wedge M(y, a) \wedge x \neq y)) \Rightarrow$
$\quad\quad \exists x, y, b.\, M(x, a) \wedge M(y, a) \wedge x \neq y \wedge T(x, y, b) \wedge F(b)$

(vi) $\forall x, y, a.\, T(x, y, a) \Rightarrow M(x, a)$

[12 marks]

(b) Is it possible to satisfy (i)–(vi) simultaneously? Either give a concrete definition of two sets I and S and relations M, F, and T for which (i)–(vi) are all true or prove that you can derive a contradiction from (i)–(vi).

[4 marks]

(c) Here are several attempts to formalise "Somebody talks about everything". Explain what they actually mean, discussing whether or not each is a reasonable formalisation.

(i) $\exists x.\forall a.\exists y.\, T(x, y, a)$

(ii) $\exists x.\exists y.\forall a.\, T(x, y, a)$

(iii) $\forall x.\forall a.\exists y.\, T(x, y, a)$

(iv) $\exists y.\forall a.\forall x.\, T(x, y, a)$

[4 marks]