COMPUTER SCIENCE TRIPOS Part IA

NATURAL SCIENCES TRIPOS Part IA (Paper CS/1)

POLITICS, PSYCHOLOGY, AND SOCIOLOGY TRIPOS Part I (Paper 7)

Monday 31 May 2010 1.30 to 4.30

COMPUTER SCIENCE Paper 1

*Answer **one** question from each of Sections A, B and C, and **two** questions from Section D.*

*Submit the answers in five **separate** bundles, each with its own cover sheet. On each cover sheet, write the numbers of **all** attempted questions, and circle the number of the question attached.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

STATIONERY REQUIREMENTS
Script paper
Blue cover sheets
Tags

SPECIAL REQUIREMENTS
Approved calculator permitted

SECTION A

1 Foundations of Computer Science

(*a*) Give an ML datatype declaration suitable for representing lazy lists, possibly of infinite length. [2 marks]

(*b*) Code the ML function `interleave`, which takes two lazy lists and generates a lazy list containing each of their elements. [2 marks]

(*c*) Code an ML function that applies a given function to every element of a lazy list, returning a lazy list of the results (analogously to the function `map`). [3 marks]

(*d*) Code the ML function `iterates` which, given a function f and some value x, generates a lazy list containing all the values of the form $f^n(x)$ (that is, $f(\cdots f(x)\cdots)$ with n applications of f) for $n \geq 0$. [3 marks]

(*e*) Code the ML function `iterates2` which, given functions f and g and values x and y, generates a lazy list containing all the values of the form $(f^m(x), g^n(y))$ for m, $n \geq 0$. [10 marks]

All ML code must be explained clearly and should be free of needless complexity.

2 Foundations of Computer Science

(*a*) Write brief notes on the function defined below:

```
fun foldl f (e, [])    = e
  | foldl f (e, x::xs) = foldl f (f(e,x), xs);
```

Illustrate your answer by describing the computations performed by the following two functions:

```
fun f x = foldl (foldl op* ) (1,x);

fun g p zs = foldl (fn ((x,y), z) =>
                        if p z then (z::x,y) else (x,z::y))
                   (([],[]), zs);
```

[4 marks]

(*b*) Selection sort is a sorting algorithm that works by repeatedly identifying and setting aside the smallest (or largest) item to be sorted. Implement selection sort in ML and describe the efficiency of your solution using O-notation. [4 marks]

(*c*) Code an ML function to generate a multiplication table in the form of a list of lists of integers. For example, given the argument 3 it should return `[[1, 2, 3], [2, 4, 6], [3, 6, 9]]`. [6 marks]

(*d*) Modify your solution to part (*c*) in order to generate a three-dimensional table containing values x_{ijk} computed by calling a supplied 3-argument curried function f. For example, given the argument 2 it should return `[[[`x_{111}`,`x_{112}`],` `[`x_{121}`,`x_{122}`]],` `[[`x_{211}`,`x_{212}`],` `[`x_{221}`,`x_{222}`]]]`. [6 marks]

All ML code must be explained clearly and should be free of needless complexity.

SECTION B

3 Discrete Mathematics I

Give structured proofs or counterexamples for the following.

(*a*) $((A \Rightarrow B) \wedge (C \Rightarrow \neg B)) \Rightarrow (A \Rightarrow \neg C)$ [10 marks]

(*b*) $(\exists x.A(x) \wedge \forall y.A(y) \Rightarrow y = x) \Rightarrow (\exists x.\forall y.(A(y) \Leftrightarrow y = x))$ [10 marks]

4 Discrete Mathematics I

Let x, y, z range over individuals I and a, b range over societies S. Let M, F and T be atomic predicates as follows:

$M(x, a)$	x is a member of society a
$F(a)$	society a involves fighting
$T(x, y, a)$	x talks to y about a

(*a*) Formalise each of the following English statements and translate each of the following formulae into idiomatic English (natural English sentences).

(*i*) $\forall x, y, a. T(x, y, a) \Rightarrow T(y, x, a)$

(*ii*) Nobody talks to themselves about anything.

(*iii*) There's at most one society involving fighting.

(*iv*) All societies have at least two members.

(*v*) $\forall a.(\exists x, y.(M(x, a) \wedge M(y, a) \wedge x \neq y)) \Rightarrow$
$\exists x, y, b. M(x, a) \wedge M(y, a) \wedge x \neq y \wedge T(x, y, b) \wedge F(b)$

(*vi*) $\forall x, y, a. T(x, y, a) \Rightarrow M(x, a)$

[12 marks]

(*b*) Is it possible to satisfy (*i*)–(*vi*) simultaneously? Either give a concrete definition of two sets I and S and relations M, F, and T for which (*i*)–(*vi*) are all true or prove that you can derive a contradiction from (*i*)–(*vi*).

[4 marks]

(*c*) Here are several attempts to formalise "Somebody talks about everything". Explain what they actually mean, discussing whether or not each is a reasonable formalisation.

(*i*) $\exists x. \forall a. \exists y. T(x, y, a)$

(*ii*) $\exists x. \exists y. \forall a. T(x, y, a)$

(*iii*) $\forall x. \forall a. \exists y. T(x, y, a)$

(*iv*) $\exists y. \forall a. \forall x. T(x, y, a)$

[4 marks]

SECTION C

5 Algorithms I

(*a*) Describe the basic principle of the *mergesort* algorithm. Illustrate your answer by showing the steps involved in sorting the array { 9, 3, 6, 2, 4, 1, 5 }.
[6 marks]

(*b*) *Insertion sort* can be considered as a mergesort where each step divides an array of size n into two arrays: one of size 1 (the element to be inserted) and one of size $(n-1)$ for array length n. By solving an appropriate recurrence relation, show that this recursive version of insertion sort has a time complexity of $O(n^2)$. Assume the time complexity for merging two arrays is $O(n)$.
[5 marks]

(*c*) A programmer is tasked with sorting both arrays and linked lists. For both data structures, he intends to use the mergesort algorithm.

(*i*) Show that the time complexity of a linked list mergesort is $O(n \log n)$. Show also that the space complexity is $O(1)$, taking care to demonstrate how this can be achieved. [6 marks]

(*ii*) The programmer only knows how to merge two arrays in $O(n)$ space and linked lists in $O(1)$ space, so proposes converting the arrays to linked lists before applying the mergesort algorithm to save on space. Comment on this strategy. [3 marks]

6 Algorithms I

The product of two $n \times n$ matrices, A and B, is a third $n \times n$ matrix, Z, where

$$Z_{ij} = \sum_{k=1}^{n} A_{ik} B_{kj}$$

(*a*) A programmer directly implements this formula when writing a function to multiply two matrices. Find the asymptotic time complexity of such an algorithm, taking care to justify your answer. [3 marks]

(*b*) An alternative strategy to compute Z is to divide A and B into four $\frac{n}{2} \times \frac{n}{2}$ matrices. Computing Z then involves eight $\frac{n}{2} \times \frac{n}{2}$ matrix multiplications followed by a series of matrix additions. This approach is then applied recursively.

(*i*) Identify the algorithmic strategy in use. [1 mark]

(*ii*) Show that the run time of this alternative strategy is given by the recurrence relation

$$t_n = K t_{\lceil f(n) \rceil} + O(g(n))$$

where t_n is the time to compute the product of two $n \times n$ matrices, K is a constant you should determine, and $f(n)$ and $g(n)$ are functions that you should identify. [6 marks]

(*iii*) Solve the recurrence relation to find an asymptotic complexity for t_n. [5 marks]

(*c*) An optimisation of the algorithm presented in part (*b*) means that only seven matrix multiplications are needed rather than the eight previously suggested. State the new recurrence relation and solve it to show that this algorithm is $O(n^{\log_2 7})$. [5 marks]

SECTION D

7 Object-Oriented Programming with Java

(*a*) Give **three** differences between an interface and an abstract class in Java. [3 marks]

(*b*) A novice programmer writes the following code in order to be able to completely clone an object of type `Car`.

```
public class Tyre {
   private int treadRemaining;

   public void SetTread(int t) { treadRemaining=t; }
   public int  GetTread() { return treadRemaining; }
}

public class Car extends Vehicle implements Cloneable {

   private Tyre tyres[] = new Tyre[4];

   public Car() {
     for (int i=0; i<4; i++) tyres[i] = new Tyre();
   }
   public Object clone() throws CloneNotSupportedException {
      Car c = new Car();
      c.tyres = this.tyres;
      return c;
   }
}
```

(*i*) Explain what it means for the `treadRemaining` field to be `private`. Explain why it is good programming practice for such fields to be `private`. [3 marks]

(*ii*) Identify the type of interface that `Cloneable` is. What is the defining characteristic of such interfaces? [2 marks]

(*iii*) Identify and explain **two** reasons why this code may not function as intended. [4 marks]

(*iv*) Rewrite the code to address the problems you have identified and allow `Car` objects to be fully cloned. [8 marks]

8 Software Design

You are required to present an outline design of a system that will be used by doctors at a medical practice to keep basic patient records. The system should record each consultation between a doctor and patient, any illness diagnosed by the doctor, and any drugs prescribed to the patient. At regular intervals, an auditor will use the system to check whether the same drug is being prescribed repeatedly to a particular patient.

(*a*) Sketch a UML use case diagram for the above functionality. [3 marks]

(*b*) Sketch a UML class diagram for a system architecture to support this functionality. [6 marks]

(*c*) Provide outline Java class and method declarations for **two** of the classes that were included in part (*b*). [4 marks]

(*d*) Sketch a UML interaction diagram showing the operation of the audit function. [3 marks]

(*e*) Describe **two** variable roles that would be required when implementing the audit function in Java. [4 marks]

9 Floating-Point Computation

(*a*) Describe the 64-bit ("double") IEEE floating-point format, including special values. [5 marks]

(*b*) Explain the following terms:

(*i*) absolute error;

(*ii*) relative error;

(*iii*) rounding error;

(*iv*) truncation error;

(*v*) machine epsilon. [5 marks]

(*c*) Outline how the implementation of the IEEE basic operations (+, −, ∗, /) are defined and their error properties. [5 marks]

(*d*) The Taylor series for cosine converges for all values of x:

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

Discuss issues in implementing a general-purpose library function that returns the value of cosine where the argument and result is a floating-point value. [5 marks]

END OF PAPER