

2009 Paper 9 Question 4

Computer Systems Modelling

Consider the birth death process model for a $M/M/1$ queue with arrival rate $\lambda > 0$, service rate $\mu > 0$ such that the traffic intensity $\rho = \lambda/\mu < 1$. Let p_k for $k = 0, 1, 2, \dots$ be the equilibrium distribution for the number of jobs, k , in the queue.

- (a) By considering transitions into and out of a given state i construct the *global balance* conditions for the equilibrium distribution. [5 marks]
- (b) By considering the transitions between a pair of adjacent states i and $i + 1$ construct the *detailed balance* conditions for the equilibrium distribution. [5 marks]
- (c) Solve the detailed balance conditions to derive the equilibrium distribution when $\rho < 1$. [5 marks]
- (d) Show that your solution for the equilibrium distribution derived in part (c) also solves your global balance conditions given in part (a). [5 marks]