2009 Paper 6 Question 4

Computation Theory

(*a*) Define what it means for a subset $S \subseteq \mathbb{N}$ to be a *recursively enumerable* set of numbers. [2 marks]

(*b*) Show that if S and S' are recursively enumerable sets of numbers, then so are the following sets (where $\langle x, y \rangle = 2^x(2y+1) - 1$).

(*i*) $S_1 = \{x \mid x \in S \text{ or } x \in S'\}$

(*ii*) $S_2 = \{\langle x, x' \rangle \mid x \in S \text{ and } x' \in S'\}$

(*iii*) $S_3 = \{x \mid \langle x, x' \rangle \in S \text{ for some } x' \in \mathbb{N}\}$

(*iv*) $S_4 = \{x \mid x \in S \text{ and } x \in S'\}$

Any standard results about partial recursive functions you use should be clearly stated, but need not be proved. [16 marks]

(*c*) Give an example of a subset $S \subseteq \mathbb{N}$ that is not recursively enumerable. [2 marks]