

2008 Paper 9 Question 13

Types

- (a) Give an account of the *Curry–Howard correspondence* between the polymorphic lambda calculus (PLC) and the second-order intuitionistic propositional calculus (2IPC). Illustrate your answer by giving a proof in 2IPC of

$$\{\} \vdash \forall p, q, r ((p \rightarrow r) \rightarrow (q \rightarrow r) \rightarrow (p \vee q) \rightarrow r)$$

corresponding to the closed PLC expression

$$\Lambda p, q, r (\lambda x : p \rightarrow r, y : q \rightarrow r, z : p \vee q (z r x y)).$$

Here $p \vee q$ is an abbreviation for $\forall r ((p \rightarrow r) \rightarrow (q \rightarrow r) \rightarrow r)$. [15 marks]

- (b) Explain how β -reduction on PLC expressions can be used to simplify proofs in 2IPC. [5 marks]