2008 Paper 9 Question 12

Computer Systems Modelling

(*a*) Suppose that you conduct a simulation experiment to estimate the mean μ of some random variable X. Supposing that your simulation experiment yields a sample of size n of independent and identically distributed values X_i derive a $100(1-\alpha)$ percent confidence interval for the parameter μ. [6 marks]

(*b*) Explain how you can use your confidence interval derived in part (*a*) to construct a rule for determining the length of your simulation so as to ensure a given size of confidence interval for the parameter μ. [4 marks]

(*c*) Now suppose that in your simulation you can also observe a second random variable Y, say, with *known* mean value μ_Y. Show that

$$E(X + c(Y - \mu_Y)) = \mu$$

where c is any constant value. [4 marks]

(*d*) Using Y as a *control variate* for X, determine the best choice of c to minimise the variance of $Z = X + c(Y - \mu_Y)$. [6 marks]