

2008 Paper 5 Question 11

Semantics of Programming Languages

Below is a fragment of L_1 , equipped with a rather strange semantics. Call it $L_?$.

Booleans $b \in \mathbb{B} = \{\mathbf{true}, \mathbf{false}\}$

Integers $n \in \mathbb{Z} = \{\dots, -1, 0, 1, \dots\}$

Locations $\ell \in \mathbb{L} = \{\ell_0, \ell_1, \ell_2, \dots\}$

Stores s , finite partial functions from \mathbb{L} to \mathbb{Z}

Operations $op ::= + \mid \geq$

Expressions $e ::= b \mid n \mid !\ell \mid \ell := e \mid e_1 \ op \ e_2 \mid \mathbf{if} \ e \ \mathbf{then} \ e_1 \ \mathbf{else} \ e_2$

(op +) $\langle n_1 + n_2, s \rangle \longrightarrow \langle n, s \rangle$ if $n = n_1 + n_2$

(op \geq) $\langle n_1 \geq n_2, s \rangle \longrightarrow \langle b, s \rangle$ if $b = (n_1 \geq n_2)$

(opA) $\frac{\langle e_1, s \rangle \longrightarrow \langle e'_1, s' \rangle}{\langle e_1 \ op \ e_2, s \rangle \longrightarrow \langle e'_1 \ op \ e_2, s' \rangle}$ (opB) $\frac{\langle e_2, s \rangle \longrightarrow \langle e'_2, s' \rangle}{\langle e_1 \ op \ e_2, s \rangle \longrightarrow \langle e_1 \ op \ e'_2, s' \rangle}$

(deref) $\langle !\ell, s \rangle \longrightarrow \langle n, s \rangle$ if $\ell \in \text{dom}(s)$ and $s(\ell) = n$

(assignA) $\langle \ell := n, s \rangle \longrightarrow \langle n, s + \{\ell \mapsto n\} \rangle$ if $\ell \in \text{dom}(s)$

(assignB) $\frac{\langle e, s \rangle \longrightarrow \langle e', s' \rangle}{\langle \ell := e, s \rangle \longrightarrow \langle \ell := e', s' \rangle}$

(ifA) $\frac{\langle e, s_1 \rangle \longrightarrow^* \langle \mathbf{true}, s_2 \rangle}{\langle \mathbf{if} \ e \ \mathbf{then} \ e_1 \ \mathbf{else} \ e_2, s_1 \rangle \longrightarrow \langle e_1, s_1 \rangle}$

(ifB) $\frac{\langle e, s_1 \rangle \longrightarrow^* \langle \mathbf{false}, s_2 \rangle}{\langle \mathbf{if} \ e \ \mathbf{then} \ e_1 \ \mathbf{else} \ e_2, s_1 \rangle \longrightarrow \langle e_2, s_1 \rangle}$

Here \longrightarrow^* is the reflexive transitive closure of \longrightarrow , defined by:

(incl) $\frac{\langle e, s \rangle \longrightarrow \langle e', s' \rangle}{\langle e, s \rangle \longrightarrow^* \langle e', s' \rangle}$ (tran) $\frac{\langle e, s \rangle \longrightarrow^* \langle e', s' \rangle \longrightarrow^* \langle e'', s'' \rangle}{\langle e, s \rangle \longrightarrow^* \langle e'', s'' \rangle}$

(refl) $\frac{}{\langle e, s \rangle \longrightarrow^* \langle e, s \rangle}$

- (a) Give a terminating sequence of reduction steps, with full derivations for each, of the configuration

$$\langle \mathbf{if} \ (\ell_0 := 3) \geq 2 \ \mathbf{then} \ 7 \ \mathbf{else} \ 8, \{\ell_0 \mapsto 0\} \rangle \quad [5 \text{ marks}]$$

- (b) Describe, with examples and alternative reduction rules, how the behaviour of $L_?$ expressions differs from that in L_1 : for (i) binary operations, (ii) store operations, and (iii) conditionals. Discuss the effects that these differences could have on programming in the language. [15 marks]