2008 Paper 13 Question 9

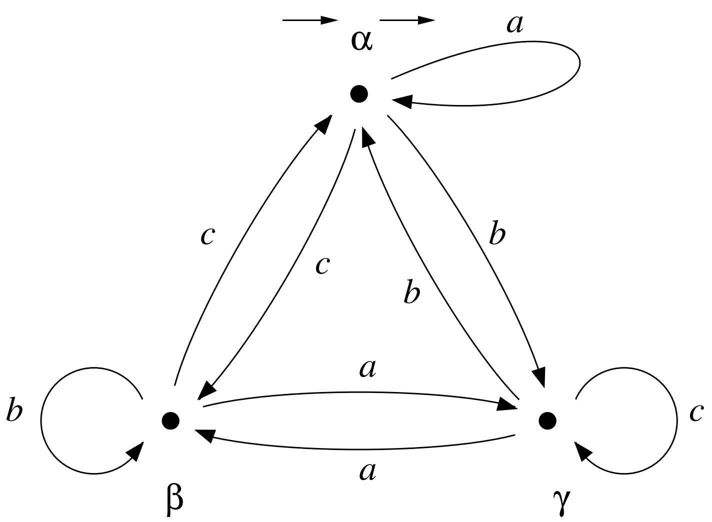
Mathematics for Computation Theory

(*a*) Prove Arden's Rule for events, that $X = A^*B$ is the least solution of the inequality $X \geqslant B + AX$. [5 marks]

(*b*) Let $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ be a partitioning of an $(n \times n)$ event matrix. You may assume that, with the same partitioning, the matrix

$$M^* \quad = \quad \begin{pmatrix} (A + BD^*C)^* & A^*B(D + CA^*B)^* \\ D^*C(A + BD^*C)^* & (D + CA^*B)^* \end{pmatrix}$$

The deterministic finite automaton M has a 3-symbol alphabet $\{a, b, c\}$, and a single accepting state α, the initial state. The transition diagram is as follows:



Show that the event accepted by M can be denoted by the regular expression

$$\{\ a + (c + bc^*a)(b + ac^*a)^*c + (b + cb^*a)(c + ab^*a)^*b\ \}^*$$

[12 marks]

Explain, with reference to M, what is happening in the term $(c + bc^*a)(b + ac^*a)^*c$ in the brackets above. [3 marks]