2008 Paper 12 Question 9

Mathematics for Computation Theory

(*a*) Let $f : A \to B$ be a function with domain A and range B. Show that the relation R on A defined by

$$(x, y) \in R \quad \Leftrightarrow \quad f(x) = f(y)$$

is an equivalence relation. [4 marks]

(*b*) A *partition* of a set A is a set $\mathcal{A}$ of disjoint subsets of A such that $A = \bigcup \mathcal{A}$, *and*

$$B, C \in \mathcal{A} \quad \Rightarrow \quad (B = C) \quad \vee \quad (B \cap C) = \emptyset$$

Let $g(n, r)$ be the number of partitions of a set A having n elements into r subsets, where $1 \leqslant r \leqslant n$. If $1 < r < n$, show that

$$g(n, r) \quad = \quad r\ g(n-1, r) + g(n-1, r-1). \qquad [7 \text{ marks}]$$

(*c*) Using the above formula, or otherwise, evaluate $g(n, r)$ in the cases:

(*i*) $r = 2$ [4 marks]

(*ii*) $r = (n - 1)$ [5 marks]