

2008 Paper 12 Question 12

Complexity Theory

(a) Give a precise definition of what it means for one decision problem to be polynomial-time reducible to another. [3 marks]

(b) Consider the following two decision problems:

HamCycle: Given a graph $G = (V, E)$ does it contain a cycle that visits every vertex exactly once?

HamPath: Given a graph $G = (V, E)$ and two distinguished vertices $s, t \in V$, is there a simple path in G that starts at s , ends at t and visits every other vertex exactly once?

Show that **HamCycle** is polynomial-time reducible to **HamPath**. [8 marks]

(c) The following decision problem is known to be solvable in polynomial time:

EulerCycle: Given a graph $G = (V, E)$ does it contain a cycle that visits every edge exactly once?

What can you conclude about the truth of the following statements? Justify your answers.

(i) **EulerCycle** is polynomial-time reducible to **HamCycle**. [3 marks]

(ii) **EulerCycle** is polynomial-time reducible to **HamPath**. [3 marks]

(iii) **HamPath** is polynomial-time reducible to **EulerCycle**. [3 marks]