2008 Paper 11 Question 8

Algorithms II

(*a*) Briefly describe the Dijkstra algorithm for finding shortest paths from a single source and explain why it cannot be used on graphs with negative edge weights. [Pseudocode is not required.] [4 marks]

(*b*) Describe and explain in detail the Johnson algorithm that finds all-pairs shortest-paths by repeatedly applying Dijkstra to each vertex, even in graphs with negative edge weights. [Pseudocode is not required but all phases of the algorithm must be clearly explained.] [7 marks]

(*c*) Some people wonder why it would not be simpler to reweight edges by adding a sufficiently large constant K to each edge weight so as to make all the weights positive. Prove that this cannot work. [2 marks]

(*d*) In Johnson's algorithm, why do we introduce a new vertex s? Could we not use, instead of a new vertex, one of the vertices of the original graph? Either prove that we can or provide a counterexample. [7 marks]