2008 Paper 11 Question 4

Introduction to Functional Programming

(a) Specify the types of the following SML functions.

(i) `fun B x y z = x (y z)` [2 marks]

(ii) `fun C x y z = x z y` [2 marks]

(iii) `fun W x y = x y y` [2 marks]

(b) Let `datatype` α `tree = leaf | node of` α `*` α `tree *` α `tree` be the datatype of binary trees.

Write an SML function `DF:` α `tree` $\rightarrow$ `int tree` that given a tree outputs a tree of the same shape, but with the values at the nodes replaced by their number in depth-first order.

For example, the depth-first numbering of the tree

```
node( "a" ,
      node( "b" , node("c",leaf,leaf) , node("c",leaf,leaf) ) ,
      node( "b" , leaf , node("c",leaf,leaf) ) )
```

is the tree

```
node( 1 ,
      node( 2 , node(3,leaf,leaf) , node(4,leaf,leaf) ) ,
      node( 5 , leaf , node(6,leaf,leaf) ) )
```

[6 marks]

(c) Let `datatype` α `inftree = node of` α `* (unit` $\rightarrow$ α `inftree list)` be the datatype of finite and infinite non-empty finitely-branching trees.

(i) The computation tree of a function $f : \alpha \rightarrow \alpha$ `list` starting at $s : \alpha$ is the possibly infinite tree with root s in which every node n has children $n_1, \ldots, n_k$ whenever $f(n) = [n_1, \ldots, n_k]$.

Write an SML function `CT:` $(\alpha \rightarrow \alpha$ `list)` $\rightarrow \alpha \rightarrow \alpha$ `inftree` such that `CT` f s is the computation tree of f starting at s. [4 marks]

(ii) Define the datatype α `seq` of finite and infinite lists of type α and write an SML function `BF:` α `inftree` $\rightarrow \alpha$ `seq` that lists the nodes of a tree according to a breadth-first traversal. [4 marks]