# 2006 Paper 4 Question 6

**Mathematical Methods for Computer Science**

Consider the $N$-point Discrete Fourier Transform (DFT) of the sequence $f[n]$ for $n = 0, 1, \ldots, N-1$ given by

$$F[k] = \sum_{n=0}^{N-1} f[n] e^{-2\pi i n k/N}$$

with the inverse DFT given by

$$f[n] = \frac{1}{N} \sum_{k=0}^{N-1} F[k] e^{2\pi i n k/N} .$$

($a$) Show that $F[k]$ has period $N$, that is $F[k+N] = F[k]$. [2 marks]

($b$) Derive the shift property in the $n$-domain, namely, that $f[n-m]$ has $N$-point DFT given by $e^{-2\pi i m k/N} F[k]$. [4 marks]

($c$) Define the $N$-point cyclic convolution, $(f \star g)[n]$, of two sequences $f[n]$ and $g[n]$ by

$$(f \star g)[n] = \sum_{m=0}^{N-1} f[m] g[n-m]$$

and show that $(f \star g)[n]$ has $N$-point DFT given by $F[k]G[k]$ where $G[k]$ is the $N$-point DFT of the sequence $g[n]$. [6 marks]

($d$) Consider the sequence $f[0] = -1, f[1] = f[2] = f[3] = 1$. Find the 4-point cyclic convolution $f \star f$ by

($i$) direct calculation; [4 marks]

($ii$) by first writing $f[n]$ in the form $f[n] = c[n] - 2d[n]$ where $c[0] = c[1] = c[2] = c[3] = 1$ and $d[0] = 1, d[1] = d[2] = d[3] = 0$. [4 marks]

You may assume that the binary operation $\star$ distributes over pointwise addition of sequences.