2006 Paper 11 Question 8

Mathematics for Computation Theory

(*a*) What is a *deterministic finite automaton* (DFA) over the finite alphabet Σ? [2 marks]

(*b*) Define the *event E accepted by* the DFA M over Σ. [3 marks]

(*c*) State Kleene's Theorem, which characterises the algebraic structure of events that are accepted by some DFA. [3 marks]

(*d*) Design a DFA over $\Sigma = \{a, b\}$ that accepts precisely those strings ending in b that do not contain two successive occurrences of a. [4 marks]

(*e*) Give an algebraic specification of the event, proving that your expression has the required properties. [8 marks]

[You may if you wish assume that if $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ is a partitioning of the transition matrix of a DFA so that A and D are square, then

$$M^* = \begin{pmatrix} (A + BD^*C)^* & A^*B(D + CA^*B)^* \\ D^*C(A + BD^*C)^* & (D + CA^*B)^* \end{pmatrix}$$

with the same partitioning.]