2006 Paper 11 Question 7

Numerical Analysis I

(a) The Newton–Raphson iteration for solution of $f(x) = 0$ is

$$\tilde{x} = x - \frac{f(x)}{f'(x)}.$$

By drawing a carefully labelled graph, explain the graphical interpretation of this formula. What is the order of convergence? [4 marks]

(b) Consider $f(x) = x^3 + x^2 - 2$. The following table shows successive iterations for each of the three starting values (i) $x = 1.5$, (ii) $x = 0.2$, (iii) $x = -0.5$. Note that, to the accuracy shown, each iteration finds the root at $x = 1$.

n	(i)	(ii)	(iii)
0	1.50000×10^{0}	2.00000×10^{-1}	-5.00000×10^{-1}
1	1.12821×10^{0}	3.95384×10^{0}	-8.00000×10^{0}
2	1.01152×10^{0}	2.57730×10^{0}	-5.44318×10^{0}
3	1.00010×10^{0}	1.70966×10^{0}	-3.72976×10^{0}
4	1.00000×10^{0}	1.22393×10^{0}	-2.56345×10^{0}
5	1.00000×10^{0}	1.03212×10^{0}	-1.72202×10^{0}
6		1.00079×10^{0}	-9.62478×10^{-1}
7		1.00000×10^{0}	1.33836×10^{0}
8		1.00000×10^{0}	1.06651×10^{0}
9			1.00329×10^{0}
10			1.00000×10^{0}
11			1.00000×10^{0}

Sketch the graph of $f(x)$ and show the first iteration for cases (i) and (ii) to show why (i) converges faster than (ii). In a separate sketch, show the first two iterations for case (iii). [Hint: a very rough sketch will suffice for case (iii).] [10 marks]

(c) Now consider $f(x) = x^4 - 3x^2 - 2$. Calculate two Newton–Raphson iterations from the starting value $x = 1$. Comment on the prospects for convergence in this case. [6 marks]