2005 Paper 9 Question 16

Topics in Concurrency

The syntax of parallel commands is given by:

$$c ::= X := a \mid c_0; c_1 \mid c_0 \parallel c_1 \mid \textbf{if } b \textbf{ then } c \mid \textbf{while } b \textbf{ do } c$$

where X ranges over locations, a over arithmetic expressions, and b over boolean expressions.

(*a*) Give an operational semantics to parallel commands, assuming an operational semantics for arithmetic and boolean expressions. [5 marks]

(*b*) This part is concerned with a Petri net semantics for parallel commands.

There are to be two kinds of conditions: *data conditions*, pairs of locations and integers, which specify the contents of locations, and *control conditions*, which specify the local control points in parallel components of commands.

A parallel command is to be represented by a basic net (where every condition has capacity one) in which a subset of control conditions I is to be distinguished as its *initial* conditions and another subset T is to be distinguished as its *terminal* conditions; the initial conditions are precisely those control conditions which hold at the start of execution of the command; the terminal conditions are precisely those control conditions which hold if and when the command terminates.

A diagrammatic account suffices for answers to the questions below.

(*i*) Describe an (infinite) net for $X := X + 1$. [2 marks]

(*ii*) Describe a construction on nets for $c_0; c_1$. [Hint: Replace the terminal conditions T_0 of c_0 and the initial conditions I_1 of c_1 with their product $T_0 \times I_1$.] [4 marks]

(*iii*) Describe a construction on nets for $c_0 \parallel c_1$. [2 marks]

(*iv*) Describe a construction on nets for **if** $X > 0$ **then** c. [2 marks]

(*v*) Describe a construction on nets for **while** $X > 0$ **do** c. [5 marks]