

## 2005 Paper 1 Question 7

### Discrete Mathematics

- (a) State and prove the Chinese Remainder Theorem concerning the simultaneous solution of a pair of congruences to co-prime moduli, and the uniqueness of that solution. [10 marks]
- (b) Define  $U_n$  (the set of units modulo  $n$ ) and  $\varphi(n)$  (Euler's totient function). [2 marks]
- (c) Given natural numbers  $m$  and  $n$  with no common factors, define  $f: U_{mn} \rightarrow U_m \times U_n$  by  $f(u) = (u \bmod m, u \bmod n)$ . Prove carefully that  $f$  is a bijective function. [6 marks]
- (d) Deduce that  $\varphi$  is multiplicative, and calculate  $\varphi(175)$ . [2 marks]