2004 Paper 5 Question 9

Logic and Proof

In this question x, y, z are variables, and a, b, c are constants.

(*a*) Briefly outline the semantics of first order logic. [5 marks]

(*b*) Use the semantics of first order logic to justify that the set of formulae

$$\{\forall x(x = c),\ P(a),\ \neg P(b)\}$$

is unsatisfiable. [2 marks]

(*c*) For each of the following first order logic formulae: **either** prove it to be valid using the sequent calculus; **or** give an interpretation that makes it false.

$$[\forall x(\exists y(R(x, y)))] \to \exists x(R(x, x))$$

$$[\exists x(\neg P(x))] \to \neg\exists x(P(x))$$

$$[\neg\exists x(P(x))] \to \exists x(\neg P(x))$$

$$\exists x(P(x) \to P(a) \wedge P(b))$$

[2 marks each]

(*d*) Consider the following set Γ of first order logic formulae:

$$\left\{\begin{array}{l} \forall x(\neg R(x, x)), \quad \forall xyz(R(x, y) \wedge R(y, z) \to R(x, z)), \\ R(a, b), \quad \forall xy(R(x, y) \to \exists z(R(x, z) \wedge R(z, y))) \end{array}\right\}$$

(*i*) Find an interpretation that satisfies Γ. [3 marks]

(*ii*) Can Γ be satisfied by an interpretation with a finite domain? Briefly justify your answer. [2 marks]