2004 Paper 5 Question 12

Complexity Theory

Recall that a *simple path* in a graph is a path with no repeated nodes. Consider the following two decision problems:

- Given a graph $G = (V, E)$, a positive integer k, source $s \in V$ and a target $t \in V$, is there a simple path from s to t of length *at least k*?

- Given a graph $G = (V, E)$, a positive integer k, source $s \in V$ and a target $t \in V$, is there a simple path from s to t of length *at most k*?

One of these problems is known to be in P while the other one is known to be NP-complete.

(*a*) Which of the two problems is in P and which is NP-complete?   [2 marks]

(*b*) Describe a polynomial time algorithm for the problem that is in P.   [6 marks]

(*c*) Give a proof of NP-completeness for the problem that is NP-complete. You may assume the NP-completeness of any problem, such as *Hamiltonian Cycle*, mentioned in the lecture course.   [12 marks]