2004 Paper 1 Question 8

Discrete Mathematics

Suppose that A is a finite set with a bijection: $A \to A \times A$. Calculate $|A|$.
[2 marks]

Give an example of a countably infinite set B with a bijection: $B \to B \times B$, proving the result carefully. [4 marks]

Consider the following definitions:

$$M = \{n \in \mathbb{N} \mid 2|n\}, \text{the even numbers}$$
$$O = \mathbb{N} \setminus M, \text{the odd numbers}$$
$$P = \mathcal{P}(\mathbb{N}), \text{the set of subsets of } \mathbb{N}$$
$$Q = \mathcal{P}(M)$$
$$R = \mathcal{P}(O)$$

Show that P, Q and R are uncountable, and construct a bijection: $P \to Q \times R$.
[12 marks]

Hence show that there is an uncountable set C with a bijection: $C \to C \times C$.
[2 marks]