COMPUTER SCIENCE TRIPOS Part II (General)
DIPLOMA IN COMPUTER SCIENCE

Tuesday 1 June 2004 1.30 to 4.30

Paper 11 (Paper 2 of Diploma in Computer Science)

Answer **five** *questions.*

Submit the answers in five **separate** *bundles, each with its own cover sheet. On each cover sheet, write the numbers of* **all** *attempted questions, and circle the number of the question attached.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 Foundations of Programming

(*a*) Briefly explain the relationship between the Java classes `Exception` and `RuntimeException` and say how they differ. [4 marks]

(*b*) The following program employs the `ArrayIndexOutOfBoundsException` (in the Java library) and also two user-supplied exceptions `TwoException` and `FiveException` (not shown) which are declared as extensions of class `Exception`. Explain how the program works and what output is produced. [6 marks]

```
public class ExceptionTest
 { private static int n=0;
   private static int[] a = {2,3,5,7,11,13,17};

   public static void main(String[] args)
    { while (true)
       { try
          { System.out.println(a[next()]);
          }
         catch(ArrayIndexOutOfBoundsException e) { break; }
         catch(FiveException e) { n++; continue; }
       }
    }

   private static int next() throws FiveException
    { try
       { if (n==2) throw new TwoException();
         if (n==5) throw new FiveException();
       }
      catch(TwoException e) {}
      return n++;
    }
 }
```

(*c*) For each of the three `catch`-clauses explain whether or not it could be removed without provoking a compile-time error. If so, explain what difference, if any, would occur at run time. [4 marks]

(*d*) Explain whether or not the `throws`-clause in the heading of method `next()` could be removed without provoking a compile-time error. [2 marks]

(*e*) Explain whether or not your answer to (*d*) would differ if the `FiveException` were declared as extending `RuntimeException` rather than `Exception`. [4 marks]

2 Compiler Construction

(*a*) A context-free grammar can be formally defined as a 4-tuple. Give a precise statement of what the components are. [2 marks]

(*b*) Explain the difference between a grammar and the language it generates. [2 marks]

(*c*) Explain what makes a grammar ambiguous, with reference to the grammar which may be commonly expressed as a "rule"

$$E ::= 1 \mid 2 \mid X \mid E + E \mid E * E \mid -E$$

where X is an identifier. [2 marks]

(*d*) For the "rule" in part (*c*), give a formal grammar containing this "rule" and adhering to your definition in part (*a*). [2 marks]

(*e*) Give non-ambiguous grammars each generating the same language as your grammar in part (*d*) for the cases:

(*i*) "$-$" is most tightly binding and "+" and "$*$" have equal binding power and associate to the left.

(*ii*) "$-$" is most tightly binding and "+" and "$*$" have equal binding power and associate to the right.

(*iii*) "$-$" binds more tightly than "+", but less tightly than "$*$", with "+" left-associative and "$*$" right-associative so that "$-a + -b * c * d + d$" is interpreted as "$((-a) + (-(b * (c * d)))) + d$".

[2 marks each]

(*f*) Give a simple recursive descent parser for your grammar in part (*e*)(*iii*) above which yields a value of type `ParseTree`. You may assume operations *mkplus*, *mktimes*, *mkneg* acting on type `ParseTree`. [6 marks]

3 Digital Electronics

An up–down binary counter is required. There is one control input (A) and a clock (CLK). The outputs are to be labelled D0, D1 and D2. If A=1 then the counter counts up every clock period, if A=0 it counts down. Design this counter in terms of AND, OR and XOR gates, and D flip-flops. Provide equations for the outputs and a circuit diagram of the complete system. [20 marks]

4 Data Structures and Algorithms

(*a*) Describe the structure of splay trees used to represent a set of key–value pairs. [5 marks]

(*b*) Describe how new key–value pairs are added to the tree, how the value associated with a given key can be looked up, and how to delete a pair with a given key. [5 marks]

(*c*) State without proof the attractive properties of splay trees. [4 marks]

(*d*) Describe the ternary tree structure used to hold a dictionary of key–value pairs where the keys are variable-length strings. Illustrate the mechanism by showing the structure after items with keys `MIT`, `SAD`, `MAN`, `APT`, `MUD`, `ADD`, `MAG`, `MINE`, `MIKE`, `MINT`, `AT`, `MATE` and `MINES` have been added in that order to an initially empty ternary tree. [6 marks]

5 Comparative Programming Languages

(*a*) Briefly describe the concept of coroutines as provided in BCPL, and outline the effect of the library functions `createco(f, size)`, `deleteco(cptr)`, `callco(cptr, val)`, and `cowait(val)`. [6 marks]

(*b*) Discuss the relative merits of BCPL coroutines *versus* those of threads such as provided in Java. [6 marks]

(*c*) Outline the overall design and organisation of a BCPL program to perform discrete event simulation using coroutines to implement the simulated activities. Concentrate on the design of the simulation event loop, the organisation of the priority queue and what functions you would provide to simplify the implementation of the activities. It would probably be sensible to adopt a programming style similar to that used in Simula 67. You should hold simulated time as a global (integer) variable. [8 marks]

6 Operating System Foundations

Two operating systems OS-A and OS-B offer only synchronous system calls, for example, for I/O. In addition, OS-A supports only one process per user-level address-space whereas OS-B supports multi-threaded applications.

(*a*) (*i*) Explain how an application-level runtime system or library running on OS-A can provide the user threads needed by concurrent programs. [8 marks]

(*ii*) Discuss any disadvantages of supporting a concurrent programming language in this way. [3 marks]

(*iii*) Are there any advantages of having only user threads? [1 mark]

(*b*) (*i*) Explain the *differences* from the runtime described for OS-A of a runtime for OS-B which maps user threads to kernel threads. [5 marks]

(*ii*) Are the disadvantages you discussed in part (*a*)(*ii*) overcome? Explain. [2 marks]

(*iii*) Have any problems been introduced by the use of kernel threads? [1 mark]

7 Continuous Mathematics

For non-negative integers r and s we have the orthogonality properties

$$\int_0^{2\pi} \cos(rx)\cos(sx)dx = \begin{cases} 2\pi & \text{if } r = s = 0 \\ \pi\delta_{rs} & \text{otherwise} \end{cases}$$

$$\int_0^{2\pi} \sin(rx)\sin(sx)dx = \begin{cases} 0 & \text{if } r = s = 0 \\ \pi\delta_{rs} & \text{otherwise} \end{cases}$$

$$\int_0^{2\pi} \sin(rx)\cos(sx)dx = 0 \qquad \forall \quad r, s$$

where

$$\delta_{rs} = \begin{cases} 1 & \text{if } r = s \\ 0 & \text{otherwise} \end{cases} .$$

(*a*) Derive expressions for the Fourier coefficients a_0, a_n, b_n $(n = 1, 2, \ldots)$ such that the infinite series

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$$

is the Fourier series for the function $f(x)$ in an interval of length 2π. [6 marks]

(*b*) For any fixed integer $N \geq 1$ let

$$S_N(x) = \frac{a_0}{2} + \sum_{n=1}^{N-1} (a_n \cos(nx) + b_n \sin(nx))$$

be the Fourier series for $f(x)$ truncated to the first N terms and let

$$S'_N(x) = \frac{a'_0}{2} + \sum_{n=1}^{N-1} (a'_n \cos(nx) + b'_n \sin(nx))$$

be any other Fourier series truncated to the first N terms. Show that

$$\int_0^{2\pi} (f(x) - S_N(x))\,(S_N(x) - S'_N(x))\,dx = 0\,.$$

[8 marks]

(*c*) Given the function $f(x)$ show that

$$\int_0^{2\pi} \left(f(x) - S'_N(x)\right)^2 dx$$

is minimised by the unique choice $a'_0 = a_0$, $a'_n = a_n$, $b'_n = b_n$ $(n = 1, 2, \ldots)$, that is, the Fourier series gives the best approximation to $f(x)$ using N terms in the sense of minimising the mean-squared error. [6 marks]

8 Numerical Analysis I

(*a*) The mid-point rule can be expressed in the form

$$I_n = \int_{n-\frac{1}{2}}^{n+\frac{1}{2}} f(x)dx = f(n) + e_n$$

where

$$e_n = f''(\theta_n)/24$$

for some θ_n in the interval $(n-\frac{1}{2}, n+\frac{1}{2})$. Assuming that a formula for $\int f(x)dx$ is known, and using the notation

$$S_{p,q} = \sum_{n=p}^{q} f(n) \ ,$$

describe a method for estimating the sum of a slowly convergent series $S_{1,\infty}$, by summing only the first N terms and estimating the remainder by integration. [5 marks]

(*b*) Assuming that $f''(x)$ is a positive decreasing function, derive an estimate of the error $|E_N|$ in the method. [5 marks]

(*c*) Given

$$\int \frac{dx}{(1+x)\sqrt{x}} = 2\tan^{-1}\sqrt{x}$$

illustrate the method by applying it to

$$\sum_{n=1}^{\infty} \frac{1}{(1+n)\sqrt{n}} \ .$$

Verify that $f''(x)$ is positive decreasing for large x, and estimate the integral remainder to be added to $S_{1,N}$. [6 marks]

(*d*) How large should N be to achieve an absolute error of approximately 2×10^{-15}? [You may assume $N+1 \simeq N$ for this purpose.] [4 marks]

9 Mathematics for Computation Theory

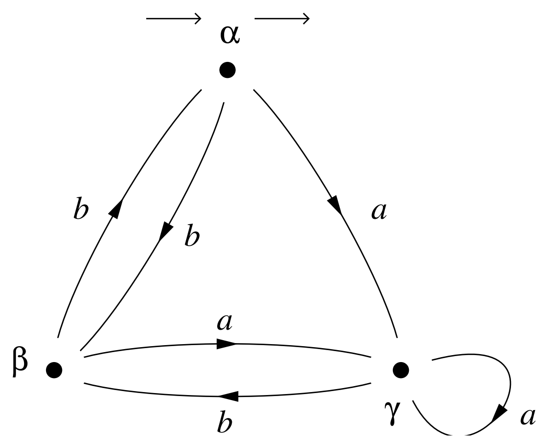
(*a*) Prove Arden's Rule for events, that $X = A^*B$ is the least solution of the inequality $X \geqslant B + AX$. [6 marks]

(*b*) Let $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ be a (2×2) event matrix. Show that the matrix

$$Y = \begin{pmatrix} (A + BD^*C)^* & A^*B(D + CA^*B)^* \\ D^*C(A + BD^*C)^* & (D + CA^*B)^* \end{pmatrix}$$

satisfies the equation $Y = I + MY$. [4 marks]

(*c*) The deterministic finite automaton M has a 2-symbol alphabet $\{a, b\}$, and a single accepting state α, the initial state. The transition diagram is as follows:



Show that the event accepted by M can be denoted by the regular expression

$$[a^*b(a\ a^*b)^*b]^*$$

[10 marks]

10 Computation Theory

(*a*) Explain what is meant by the following statements:

(*i*) $f : \mathbb{N} \to \mathbb{N}$ is a *total recursive* (TR) function; [3 marks]

(*ii*) the sequence $\{f_n : \mathbb{N} \to \mathbb{N}\}_{n \in \mathbb{N}}$ of TR functions of a single variable is recursively enumerable. [4 marks]

(*b*) Show that no recursive enumeration can include the set of *all* TR functions of a single variable. [4 marks]

(*c*) Suppose $u(n, x)$ is a recursive enumeration of the sequence of TR functions $f_n(x) = u(n, x)$. Show how to define a sequence $\{g_n : \mathbb{N} \to \mathbb{N}\}$ of TR functions of a single variable such that each g_n is distinct from every function f_n, and also from each g_k for $k \neq n$. [5 marks]

(*d*) Express the sequence $\{g_n\}$ as an explicit recursive enumeration $v(n, x) = g_n(x)$. [4 marks]

11 Software Engineering and Design

(*a*) Compiler development is sometimes cited as an example of a situation in which the waterfall model should be applied. Why is this? [2 marks]

(*b*) Imagine that a user-centred design model were applied to the development of programming language products instead. Discuss what might happen differently during the development of those products. [8 marks]

(*c*) Suggest *two* design innovations that might result from the new approach. Draw a UML diagram to illustrate *each* of those designs, using a different kind of diagram in each case. [10 marks]

12 Business Studies

A software project has two phases, each with three tasks. They are expected to take the following amount of effort:

Phase 1:	
Analysis:	3 weeks
Code:	2 weeks
Test:	1 week
Phase 2:	
Analysis:	1 week
Code:	2 weeks
Test:	3 weeks

Within a phase a task cannot start until the previous task completes. A task in Phase 2 cannot start until the corresponding task in Phase 1 has completed.

(*a*) Draw the PERT and Gantt charts for the project. What is the minimum elapsed time? [5 marks]

(*b*) Two staff are assigned to the project, an analyst and a programmer. The analyst also acts as test engineer. How long will the project take with this staffing? [5 marks]

(*c*) The analyst is charged out at a fixed rate of £500/day and the programmer at £300/day, including overheads. How much would you quote for the project and why? [5 marks]

(*d*) Explain how you would monitor such a project. How would you turn the result into a product? [5 marks]

END OF PAPER