2003 Paper 9 Question 14

Denotational Semantics

(a) State without details *three* ways to prove properties of the least fixed point $\mathit{fix}(f)$ of a continuous function f on a domain. [3 marks]

(b) Let $f : D \to D$ be a continuous function on a domain D. Explain why $\mathit{fix}(f \circ f) = \mathit{fix}(f)$. [3 marks]

(c) Let both $f : D \to D$ and $g : D \to D$ be continuous functions on a domain D. Prove

$$\mathit{fix}(f \circ g) = f(\mathit{fix}(g \circ f))$$

by showing

(i) $\mathit{fix}(f \circ g) \sqsubseteq f(\mathit{fix}(g \circ f))$, and [4 marks]

(ii) $f(\mathit{fix}(g \circ f)) \sqsubseteq \mathit{fix}(f \circ g)$. [10 marks]

[Hint: For the last part, start by expanding the left-hand side as a least upper bound of approximations.]