2003 Paper 1 Question 8

Discrete Mathematics

(a) Define the terms *injective*, *surjective* and *bijective*, and state the Schröder–Bernstein theorem concerning the existence of a bijection between two sets. [4 marks]

(b) What is a countable set? [2 marks]

(c) Prove the following assertions:

(i) If C is a countable set and $f : A \to C$ is an injection, then A is countable. [2 marks]

(ii) If A and B are countable sets, then $A \times B$ is countable. [2 marks]

(iii) $\mathbb{Z}$ and $\mathbb{Q}$, the sets of integer and rational numbers, are both countable. [4 marks]

(iv) $\mathcal{P}(\mathbb{N})$, the set of all subsets of the natural numbers, is not countable. [2 marks]

(v) If U is an uncountable set and $f : U \to V$ is an injection, then V is not countable. [2 marks]

(vi) $\mathbb{R}$, the set of real numbers, is not countable. [2 marks]