2003 Paper 11 Question 10

Computation Theory

What is the *Church–Turing Thesis*? Briefly describe some evidence that it is true.

[4 marks]

Using the Church–Turing Thesis, or otherwise, show that if $f(x)$ and $g(x)$ are partial recursive functions of a single argument, then so are the following functions, where $dom(f)$ denotes the set of integers x for which $f(x)$ is defined, and similarly for $dom(g)$.

$$h(x) = \begin{cases} x & \text{if } x \in dom(f) \text{ and } x \in dom(g) \\ \text{undefined} & \text{otherwise} \end{cases}$$

[4 marks]

$$k(x) = \begin{cases} x & \text{if } x \in dom(f) \text{ or } x \in dom(g) \\ \text{undefined} & \text{otherwise} \end{cases}$$

[6 marks]

Is the partial function defined by

$$f'(x) = \begin{cases} x & \text{if } x \notin dom(f) \\ \text{undefined} & \text{otherwise} \end{cases}$$

necessarily partial recursive if f is? Justify your answer.

[6 marks]