2002 Paper 8 Question 13

Computer Vision

The following very useful operator is often applied to an image $I(x, y)$ in computer vision algorithms, to generate a related "image" $g(x, y)$:

$$g(x,y) = \int_\alpha \int_\beta \nabla^2 e^{-\left((x-\alpha)^2+(y-\beta)^2\right)/\sigma^2} I(\alpha,\beta)\; d\alpha\; d\beta$$

where

$$\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$$

(*a*) Give the general name for this type of mathematical operator, and the chief purpose that it serves in computer vision. [2 marks]

(*b*) What image properties should correspond to the zeroes of the equation, i.e. those points (x, y) in the image $I(x, y)$ where the above result $g(x, y) = 0$? [3 marks]

(*c*) What is the significance of the parameter σ? If you increased its value, would there be more or fewer points (x, y) at which $g(x, y) = 0$? [3 marks]

(*d*) Describe the effect of the above operator in terms of the two-dimensional Fourier domain. What is the Fourier terminology for this image-domain operator? What are its general effects as a function of frequency, and as a function of orientation? [4 marks]

(*e*) If the computation of $g(x, y)$ above were to be implemented entirely by Fourier methods, would the complexity of this computation be greater or less than the image-domain operation expressed above, and why? What would be the trade-offs involved? [4 marks]

(*f*) If the image $I(x, y)$ has 2D Fourier Transform $F(u, v)$, provide an expression for $G(u, v)$, the 2D Fourier Transform of the desired result $g(x, y)$ in terms of only the Fourier plane variables $u, v, F(u, v)$, and the above parameter σ. [4 marks]