2002 Paper 12 Question 9

Numerical Analysis II

(*a*) In the Chebyshev characterisation theorem, the best L_∞ approximation to $f(x)$ over a closed finite interval by a polynomial $p_{n-1}(x)$ of degree $n-1$ has the property that the maximum error $|e(x)|$ is attained at M distinct points ξ_j such that $e(\xi_j) = -e(\xi_{j-1})$. What is M? [2 marks]

(*b*) Let $x = m \times 2^k$ represent a normalised number in a floating-point implementation. When computing $\sqrt{x}$ show how the domain of the problem can be reduced to $x \in [1, 4)$. Find the coefficients a, b which minimise $||e(x)||_\infty$ over $[1, 4]$ where $e(x) = ax + b - \sqrt{x}$. [8 marks]

(*c*) Taking full account of symmetry, describe the form of the best polynomial approximation $p(x)$ to x^4 over $[-1, 1]$ by a polynomial of lower degree. Sketch x^4 and $p(x)$, showing the extreme values of $|e(x)|$ where $e(x) = x^4 - p(x)$. Hence compute the coefficients of $p(x)$. [10 marks]