

2002 Paper 11 Question 8

Numerical Analysis I

- (a) Define *absolute error* and *relative error*. How are they related? How are absolute errors combined when two numbers are added together? How are relative errors combined when two numbers are multiplied together? [5 marks]
- (b) Explain the term *loss of significance* in terms of absolute error and relative error. [1 mark]
- (c) Writing δ_x for the relative error in x , what is the worst-case *relative error* in evaluating x^2 ? What is the worst-case *absolute error*? What is the worst-case *absolute error* in evaluating $x^2 - y^2$? [3 marks]
- (d) Let δ_s, δ_c be the relative errors in the values of $\sin \theta, \cos \theta$ respectively. Find the worst-case relative errors in evaluating each of the formulae

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

For what values of $\cos \theta$ does the second formula display loss of significance? [6 marks]

- (e) Consider the evaluation of $x^2 + y^2 - z^2$ in two cases

(i) $|y| \simeq |z|$, $|x|$ is very small,

(ii) $|x| \simeq |y| \simeq |z|$, $|x|$ is not small.

Taking each case separately, can loss of significance occur? Explain your answers. [4 marks]

- (f) How would you compute $x^2 + y^2 - z^2$ to achieve greater accuracy, especially if guard digits were in use? [1 mark]