2001 Paper 9 Question 6

Types

(*a*) Define the typing relation of the polymorphic lambda calculus (PLC). [5 marks]

(*b*) For *each* of the following PLC typing judgements, state whether or not there is a type τ that makes the judgement provable. Justify your answer in each case.

(*i*) $\vdash \lambda x : \forall\alpha(\alpha)(\Lambda\beta(x\,\beta)) : \tau$ [3 marks]

(*ii*) $\vdash \Lambda\alpha(\lambda x : \alpha(\Lambda\beta(x\,\beta))) : \tau$ [3 marks]

(*iii*) $\vdash \lambda x : \tau(\Lambda\alpha(x(\alpha \to \alpha)(x\,\alpha))) : \tau \to \forall\beta(\beta)$ [3 marks]

(*iv*) $\vdash \lambda x : \tau(\Lambda\alpha(x(\alpha \to \alpha)(x\,\alpha))) : \tau \to \forall\alpha(\alpha \to \alpha)$ [3 marks]

(*v*) $\vdash \Lambda\alpha(\lambda x : \tau(x(\alpha \to \alpha)(x\,\alpha))) : \forall\alpha(\alpha \to \alpha)$ [3 marks]