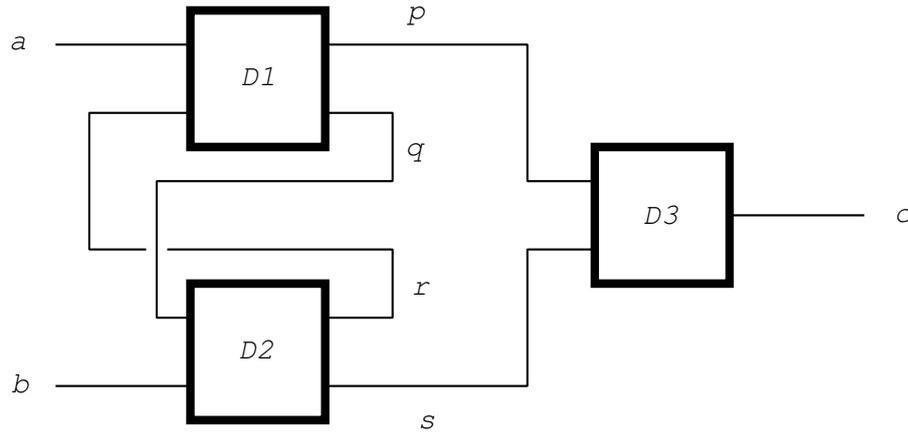## 2001 Paper 9 Question 12

Specification and Verification II

Suppose definitions of $D1(a, r, p, q)$, $D2(q, b, r, s)$ and $D3(p, s, c)$ are given.

(*a*) Using $\exists$ and $\wedge$ write down a definition of a predicate D such that $D(a, b, c)$ defines the relation between a, b and c when $D1$, $D2$ and $D3$ are connected as in the following diagram. [4 marks]

Suppose now that $D1$, $D2$ and $D3$ are defined in terms of functions f_1, f_2, f_3, f_4 and f_5 by

$$D1(a, r, p, q) = (p = f_1\ a\ r)\ \wedge\ (q = f_2\ a)$$
$$D2(q, b, r, s) = (r = f_3\ q\ b)\ \wedge\ (s = f_4\ q\ b)$$
$$D3(p, s, c) = (c = f_5\ p\ s)$$

(*b*) Write down an equation expressing c in terms of a and b. [4 marks]

(*c*) Show the logical steps needed to derive the equation from the definition of D. [12 marks]