COMPUTER SCIENCE TRIPOS Part Ib

Wednesday 6 June 2001 1.30 to 4.30

Paper 5

Answer **five** *questions.*

No more than **two** *questions from any one section are to be answered.*

Submit the answers in five **separate** *bundles, each with its own cover sheet. On each cover sheet, write the numbers of* **all** *attempted questions, and circle the number of the question attached.*

Write on **one** *side of the paper only.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 Data Structures and Algorithms

(*a*) Describe and justify an algorithm for finding the shortest distance between each pair of vertices in an undirected graph in which each edge has a given positive length. If there is no path between a pair of vertices a very large result should arise. [12 marks]

(*b*) Is it sensible to use your algorithm to discover whether such a graph is connected? Suggest an alternative that would be appropriate for a graph of 1000 vertices and 10,000 edges. [8 marks]

2 Computer Design

(*a*) What is a *pipeline bubble* and why might a branch instruction introduce one or more bubbles? [4 marks]

(*b*) Explain, with the aid of an example, how conditional instructions may be used to reduce the number of bubbles in a pipeline. [4 marks]

(*c*) What is the difference between *branches*, *interrupts*, *software interrupts* (initiated by a SWI instruction on the ARM) and *exceptions*? [8 marks]

(*d*) What is an *imprecise exception* and why might a processor designer prefer it to a precise exception mechanism? [4 marks]

3 Digital Communication I

Information is to be conveyed from A to B using automatic repeat request (ARQ), forward error correction (FEC), and lossless compression.

(a) Explain the terms *ARQ*, *FEC* and *lossless compression*. [5 marks]

(b) If we consider each of these functions to be operating at different protocol layers, what would be the most sensible ordering of the layers, and why? [5 marks]

(c) Suppose:

- The underlying bit channel has a capacity of B, a delay τ and error rate ϵ_0.
- The compression ratio is $C < 1$.
- The FEC has rate $R < 1$ and given an error rate ϵ_0 provides an error rate ϵ_1 (which is detected).
- The ARQ protocol has a window size of W.

At what rate can the information be conveyed? [Hint: Consider when retransmissions are made.] State any assumptions you make about the operation of the ARQ protocol. [10 marks]

4 Computer Graphics and Image Processing

(*a*) Describe the limitations of human vision in terms of:

(*i*) spatial resolution

(*ii*) luminance

(*iii*) colour

and explain the implications that each of these limitations has on the design of display devices. [10 marks]

(*b*) In image compression we utilise three different mechanisms to compress pixel data:

(*i*) mapping the pixel values to some other set of values

(*ii*) quantising those values

(*iii*) symbol encoding the resulting values

Explain each mechanism, describe the way in which it helps us to compress the image, and describe in what way it affects the visual quality of the resulting (decompressed) image when compared with the original. [10 marks]

SECTION B

5 Comparative Programming Languages

(*a*) Outline the main innovations that are in Simula 67 but were not in Algol 60, paying particular attention to Simula Classes. [6 marks]

(*b*) Illustrate how Simula can be used to simulate a small restaurant with six tables, two waiters and small groups of customers arriving at random intervals. You need specify only the classes you would define. Most of the algorithmic details may be omitted. [6 marks]

(*c*) Discuss to what extent Simula has been made redundant by the development of modern object-oriented languages such as Java. [8 marks]

6 Compiler Construction

Consider a language J which has

- Java-like syntax
- nested definitions of procedures within other procedures
- local variables (with static binding)
- raising and handling of named, parameterless exceptions

Explain a possible run-time data structure which a compiler for J might use. [10 marks]

A naïve user of such a language suggests that the resultant compiled code will spend a significant fraction of execution time searching—both finding the store location corresponding to the use of a variable name and finding the exception handler corresponding to the raising of a given exception name.

Determine with justification whether this is so for your run-time data-structure proposed above. [4 marks]

Now instead suppose a simple interpreter for J is written, so that searches for variable (or exception) names search the appropriate environment for the variable value or exception handler code. To what extent are these searches bounded by (a) the number of variables or exceptions in the program or (b) the dynamic or static nesting of procedures? [6 marks]

7 Prolog for Artificial Intelligence

Consider the following Prolog program, which is intended to define the third argument to be the maximum value of the first two numeric arguments:

```
max(X, Y, X) :- X >= Y, !.
max(X, Y, Y).
```

(a) Provide an appropriate query to show that the above program can give an incorrect result. [4 marks]

(b) Explain the cause of the error. [6 marks]

(c) Suggest a correction. [5 marks]

(d) Write a Prolog program to find the maximum of a list of numbers. [5 marks]

8 Databases

(*a*) Explain how to describe the structure of a collection of data using *entities*, *attributes* and *relationships*. [6 marks]

(*b*) How would you identify particular instances of data in order to record the information in a database? Illustrate your answer by considering both a relational database maintained using SQL-92 and an ODMG database. [6 marks]

(*c*) A high street bank has just announced a merger with a nationwide building society. You are employed as a consultant to advise on the integration of their client databases.

Both institutions use relational databases. Write brief notes to alert the database administrators to the difficulties that they may encounter. [8 marks]

SECTION C

9 Semantics of Programming Languages

(*a*) The integer expressions E of a programming language are given by

$$E ::= n \mid X \mid -E \mid E + E$$

where n ranges over integer constants and X ranges over identifiers. Explain the principle of *structural induction* for proving that some property $\Phi(E)$ holds for all integer expressions E. [5 marks]

(*b*) Taking *states* to be finite partial functions mapping identifiers to integer constants, define a relation

$$E, s \Downarrow n$$

giving the result n (if any) of evaluating integer expression E in state s. [7 marks]

(*c*) Use structural induction to prove that if $E, s \Downarrow n_1$ and $E, s \Downarrow n_2$ both hold, then $n_1 = n_2$. [Hint: Consider the property $\Phi(E)$ given by $\forall s, n_1, n_2((E, s \Downarrow n_1)\ \&\ (E, s \Downarrow n_2) \Rightarrow n_1 = n_2)$.] [7 marks]

(*d*) What property of the pair E, s ensures that there is some n for which $E, s \Downarrow n$ holds? [1 mark]

10 Foundations of Functional Programming

(*a*) Write a pure lambda-expression that will act as a fixed-point operator `Y` such that the identity `Y f = f(Y f)` will hold. [6 marks]

(*b*) Write pure lambda-expressions that define functions `P`, `A` and `D` such that `A (P x y) = x` and `D (P x y) = y`. Observe that `P` can be thought of as creating a 2-tuple and `A` and `D` then act as selectors that can retrieve the two components. [7 marks]

(*c*) Using the two above lambda-expressions it is possible to express mutual recursion between two functions, say `f` and `g`. This can be done by using `Y` to help find the value of `(P f g)` the tuple whose elements are `f` and `g`. Using the artificial and rather silly example [the example will never terminate since it has no stopping condition!]

```
    f x = g (f (g x))
AND g x = g (f x)
```

show how to construct a pure lambda expression that would evaluate

```
(f g)
```

[7 marks]

11 Logic and Proof

(*a*) Explain the meaning of the notation $A \models B$, where A and B denote formulae of (*i*) propositional logic and (*ii*) S4 modal logic. [2+6 marks]

(*b*) For each of the following equivalences, state whether it holds or not, justifying each answer rigorously.

$$\begin{aligned}(P \wedge (Q \to R)) \to S &\simeq (\neg P \vee \neg Q \vee S) \wedge (\neg P \vee \neg R \vee S)\\ (P \to Q) \to (Q \to P) &\simeq (Q \to P)\\ \forall xy\,(P(x) \vee \neg P(y)) &\simeq \forall xy\,(P(x) \leftrightarrow P(y))\end{aligned}$$

[3+3+6 marks]

12 Complexity Theory

(*a*) Give a precise definition of the complexity class *NP* and of *NP-completeness*. [2 marks each]

(*b*) For any natural number k, the problem **k-colourability** is defined as the following decision problem.

> Given a graph $G = (V, E)$, is there a mapping $\chi : V \to \{c_1, \ldots, c_k\}$ such that if $(u, v) \in E$, then $\chi(u) \neq \chi(v)$?

(*i*) Explain why, for each k, the problem **k-colourability** is in the class NP. [2 marks]

(*ii*) For what values of k is the problem **k-colourability** decidable in polynomial time? Why? [2 marks]

(*iii*) For which values of k is the problem **k-colourability** NP-complete? Give a brief indication how this might be proved. [2 marks]

(*c*) The company Fon-X runs a mobile 'phone service. It has 2000 'phone masts stationed across the country. The frequency spectrum assigned to the company is split into 20 bands. Each mast is to be assigned a frequency band in such a way that masts within 50 miles of each other do not share the same frequency band.

(*i*) What is the relationship between this problem and **k-colourability**? [2 marks]

(*ii*) What can you say about the complexity of the problem Fon-X is trying to solve? [3 marks]

(*d*) Fon-X solved its frequency assignment problem by an exhaustive search algorithm, which took a week to run. The company has just doubled in size through a merger. It intends to repeat the frequency assignment on 4000 masts, setting aside two weeks for the task. As a consultant, write a short note to the company explaining what you think of the idea, and suggesting any alternatives you think might be better. [5 marks]

END OF PAPER