2000 Paper 6 Question 11

Logic and Proof

For each of the given pairs of terms, give a most general unifier or indicate why none exists. (Here x, y, z are variables while a, b are constant symbols.)

$$\begin{array}{rcl} h(x,y,x) & \text{and} & h(y,z,u) \\ h(x,y,z) & \text{and} & h(f(y),z,x) \\ h(x,y,b) & \text{and} & h(a,x,y) \\ h(x,y,z) & \text{and} & h(g(y,y),g(z,z),g(u,u)) \end{array}$$

[4 marks]

A standard unification algorithm takes a pair of terms t_1 and t_2 and returns a substitution θ such that $t_1\theta = t_2\theta$. Show how this algorithm can be used to find the unifier of several ($n > 2$) terms t_1, t_2, …, t_n: a substitution θ such that $t_1\theta = t_2\theta = \cdots = t_n\theta$. Indicate how the unifier is constructed from the unifiers of $n-1$ pairs of terms. (Assume that all required unifiers exist and ignore the question of whether the unifiers are most general.) [6 marks]

Prove using resolution the formula

$$\bigl(\forall x\,[P(x) \leftrightarrow (Q(x) \land \neg Q(f(x)))]\bigr) \to \exists y\,\neg P(y)$$

or indicate why this formula is not a theorem. [10 marks]