COMPUTER SCIENCE TRIPOS Part IB

Tuesday 6 June 2000 1.30 to 4.30

Paper 4

Answer **five** *questions.*

Submit the answers in five **separate** *bundles, each with its own cover sheet. On each cover sheet, write the numbers of* **all** *attempted questions, and circle the number of the question attached.*

Write on **one** *side of the paper only.*

1 Concurrent Systems

In a proposed, next-generation banking system a number of transactions are to be scheduled to run concurrently:

- Debit (D) transactions to make payments from customer accounts to a credit card company.
- Interest (I) transactions to add daily interest to customer account balances.
- Transfer (T) transactions which first check whether the source account contains sufficient funds then either abort or continue the transfer from source to destination accounts. Customer x is running a T to transfer £1000 from A to B. Customer y is running a T to transfer £200 from B to A.

(a) Discuss the potential for interference between any of these transactions. [7 marks]

(b) Demonstrate the effect of concurrency control based on strict two-phase locking in relation to the discussion in (a). [8 marks]

(c) Comment on the scope of concurrency control in relation to the discussion in (a). [5 marks]

[Hint: you may assume that operations on bank account objects, such as debit, credit and add-interest are atomic.]

2 Continuous Mathematics

(a) In his formulation of the calculus, Newton captured only the notion of integer-order differentiation considering first-, second- and third-order derivatives, and so on. In scientific computing, however, we sometimes need fractional-order derivatives, as for example in fluid mechanics.

Explain how *Fractional Differentiation* (derivatives of non-integer order) can be given precise quantitative meaning through Fourier analysis. [5 marks]

Suppose that a continuous function $f(x)$ has Fourier Transform $F(\mu)$. Outline an algorithm (as a sequence of mathematical steps, not an actual program) for computing the 1.5^{th} derivative of some function $f(x)$

$$\frac{d^{(1.5)}f(x)}{dx^{(1.5)}}$$

[5 marks]

(b) For $i = \sqrt{-1}$, consider the quantity $\sqrt{i}$.

(i) Express $\sqrt{i}$ as a complex exponential. [1 mark]

(ii) In which quadrant of the complex plane does it lie? [1 mark]

(iii) What is the real part of $\sqrt{i}$? [1 mark]

(iv) What is the imaginary part of $\sqrt{i}$? [1 mark]

(v) What is the length (the modulus) of $\sqrt{i}$? [1 mark]

(c) Initial-value problems described by ordinary differential equations have solutions that can be propagated forward using incrementing rules such as Euler or Runge–Kutta. But boundary-value problems specified by partial differential equations (PDEs) such as Poisson's Equation,

$$\frac{\partial^2\mu(x,y)}{\partial x^2} + \frac{\partial^2\mu(x,y)}{\partial y^2} = \rho(x,y)$$

cannot be solved by such propagation methods. Why not? [3 marks]

State the principle for one general class of numerical methods for solving such PDEs. [2 marks]

3 Further Java

Describe the model for handling graphical output and interactive input in the Abstract Windowing Toolkit (AWT) for Java. Your answer should cover the use of

- hierarchies of classes
- overriding methods
- interfaces
- inner classes
- spatial hierarchy

[4 marks each]

4 Compiler Construction

Consider the grammar

```
S -> E <eof>
E -> T + E
E -> T
T -> x
```

where `S` is the starting symbol, `<eof>` is a special token marking end of input and `x` is a terminal.

Explain and find the left, right and follow sets for all non-terminals in the grammar. [5 marks]

Suppose that an SLR parser for this grammar is required. One stage on the way to constructing the parsing tables is to create the *characteristic finite state machine* (sometimes known as the LR(0) states). Do this, explaining your working clearly. You do not need to complete the SLR parsing tables. [10 marks]

Now, assuming that the parsing tables have been constructed, show what values will be placed on a stack and comment about internal state while an SLR parser using this grammar processes the input text `x+x+x<eof>`. [5 marks]

5 ECAD

A naïve Verilog programmer has made two attempts at implementing a `RandomBits` module. The module is supposed to generate a pseudo-random sequence of bits.

```
module RandomBitsA( rand_bits );
  output rand_bits;
  wire   [3:0] shift_reg;
  wire   newbit = ~(shift_reg[3] ^ shift_reg[1] ^ shift_reg[0]);
  assign shift_reg[3] = shift_reg[2];
  assign shift_reg[2] = shift_reg[1];
  assign shift_reg[1] = shift_reg[0];
  assign shift_reg[0] = newbit;
  assign rand_bits = newbit;
endmodule

module RandomBitsB( clk, rand_bits );
  input  clk;
  output rand_bits;
  reg    [3:0] shift_reg;
  wire   newbit = ~(shift_reg[3] ^ shift_reg[1] ^ shift_reg[0]);
  always @(posedge clk) begin
    shift_reg[3] <= shift_reg[2];
    shift_reg[2] <= shift_reg[1];
    shift_reg[1] <= shift_reg[0];
    shift_reg[0] <= newbit;
  end
  assign  rand_bits = newbit;
endmodule

// TestRandom is the top level module
module TestRandom( clk, tstA, tstB );
  input  clk;                  // input clock
  output tstA, tstB;           // output test bits
  RandomBitsA randa( tstA );
  RandomBitsB randb( clk, tstB );
endmodule
```

(*a*) In Verilog, what is the difference between continuous assignment and non-blocking assignment? [4 marks]

(*b*) What are the circuit diagrams corresponding to `RandomBitsA` and `RandomBitsB`? [8 marks]

(*c*) What is the output from `RandomBitsA` and `RandomBitsB`? [8 marks]

6 Data Structures and Algorithms

Describe in detail both Prim's and Kruskal's algorithms for finding a minimum cost spanning tree of an undirected graph with edges labelled with positive costs, and explain why they are correct. [7 marks each]

Compare the relative merits of the two algorithms. [6 marks]

7 Operating System Functions

Why is it important for an operating system to schedule disc requests? [4 marks]

Briefly describe each of the SSTF, SCAN and C-SCAN disc scheduling algorithms. Which problem with SSTF does SCAN seek to overcome? Which problem with SCAN does C-SCAN seek to overcome? [5 marks]

Consider a Winchester-style hard disc with 100 cylinders, 4 double-sided platters and 25 sectors per track. The following is the (time-ordered) sequence of requests for disc sectors:

$$\{\ 3518,\ 1846,\ 8924,\ 6672,\ 1590,\ 4126,\ 107,\ 9750,\ 158,\ 6621,\ 446,\ 11\ \}$$

The disc arm is currently at cylinder 10, moving towards 100. For each of SSTF, SCAN and C-SCAN, give the order in which the above requests would be serviced. [3 marks]

Which factors do the above disc arm scheduling algorithms ignore? How could these be taken into account? [4 marks]

Discuss ways in which an operating system can construct logical volumes which are (*a*) more reliable and (*b*) higher performance than the underlying hardware. [4 marks]

8 Computation Theory

Let $\mathbb{N}$ be the natural numbers $\{0, 1, 2 \ldots\}$.

What is meant by each of the following statements?

- The subset $S \subseteq \mathbb{N}$ is *recursive*.
- The subset $S \subseteq \mathbb{N}$ is *recursively enumerable*.

[5 marks]

How would you extend the definition of *recursive enumeration* to sets of computable functions? [3 marks]

A sequence of natural numbers is a total function $s : \mathbb{N} \to \mathbb{N}$. The sequence is *recursive* if and only if s is computable.

A finite sequence σ of natural numbers is specified by a pair (l, x), where $l \in \mathbb{N}$ is the number of elements, and $x : [1, l] \to \mathbb{N}$ is a function that defines those elements. The case $l = 0$ defines the null sequence.

In each of the following cases, establish whether the set defined is recursively enumerable:

(a) the set of all recursive subsets of $\mathbb{N}$ [5 marks]

(b) the set of all recursive sequences of natural numbers [2 marks]

(c) the set of all finite sequences of natural numbers [5 marks]

9 Numerical Analysis I

Define the *absolute error* ε_x and *relative error* δ_x in representing a number x. How are these errors related? Which type of error is associated with the term *loss of significance*? Define *machine epsilon* ε_m. [4 marks]

Write down expressions for the worst case errors $|\delta_{xy}|$ and $|\varepsilon_{xy}|$ in computing xy. Express your answers in terms of δ_x and δ_y. Hence find expressions for $|\varepsilon_{xy+z}|$ and $|\delta_{xy+z}|$ in terms of δ_x, δ_y and δ_z. [6 marks]

Assuming $|\,\delta_x\,| = |\,\delta_y\,| = |\,\delta_z\,| = \varepsilon_m$, find an expression for $|\,\delta_{xy+z}\,|/\varepsilon_m$. When would you expect loss of significance in computing $xy + z$? [3 marks]

The formula

$$\tilde{f}'(x) = \frac{f(x+h) - f(x)}{h}$$

is used to compute $f'(x)$ for a certain function $f(x)$ when $\varepsilon_m = 10^{-10}$. The formula is applied at $x = 0.2$ where $f(0.2) = 1.1$.

(*a*) If $h = 10^{-3}$ then $\tilde{f}'(0.2) \simeq 8.44$.

(*b*) If $h = 10^{-8}$ then $\tilde{f}'(0.2) \simeq 8.40$.

However, it is known that $f'(0.2) = 8.42$ to 3 significant digits. Why is (*a*) such a poor estimate? Why is (*b*) such a poor estimate? [4 marks]

Suggest a more suitable value for h. Roughly how many correct significant decimal digits would you expect to get in your answer? [3 marks]

10 Computer Graphics and Image Processing

Describe an algorithm for performing scan conversion of a set of 3D polygons, including details of clipping, projection, and the underlying 2D polygon scan conversion algorithm. You may assume that you are given the colour of each polygon and that no lighting calculations are required. *Ray tracing* is not an acceptable answer to this question. [20 marks]

END OF PAPER