1999 Paper 5 Question 12

Foundations of Functional Programming

Explain why the Church–Rosser theorem implies that there are λ-terms that are not equal to each other. [2 marks]

Suppose the following reduction rule is added to the λ-calculus:

$$\lambda xy.x \to \lambda xy.y$$

Show that in the resulting calculus, all terms are equal. [3 marks]

Let $A = \lambda xy.y(xxy)$ and $\Theta = AA$. Show that Θ is a fixed-point combinator. [3 marks]

Assume an encoding of lists where $[a_1, \ldots, a_m]$ is represented by

$$\lambda fx.fa_1(fa_2 \cdots (fa_m x) \cdots)$$

Use the fixed-point combinator Θ to obtain a λ-term **rev** such that:

$$\mathbf{rev}[a_1, \ldots, a_m] = [a_m, \ldots, a_1]$$

You may assume a λ-term representation of the booleans and of ordered pairs, but you should define any other terms you require. [12 marks]