1999 Paper 5 Question 11

Complexity Theory

Explain briefly, stating but not proving any relevant results, which of the following statements are true, which are false and which are meaningless in the context of a study of the complexity of computation. [*Each part will be allocated the same weight when marking, but conciseness and clarity of explanation will be important as well as simple factual correctness.*]

(*a*) I can check an integer N to see whether it is prime by doing test division by all the numbers less than it. This involves just under N trial divisions, and division has a polynomial cost. Therefore testing to see whether a number is prime is a problem in the class P.

(*b*) If I am given an integer N and am told in advance that it is composite then I can guess a pair of integers P and Q, multiply them together and check whether their product is N. Multiplication has polynomial cost hence factorising known-composites is in the class NP.

(*c*) The quotient of a pair of n-digit integers can be computed in a time less than $kn^{1.1}$ for some value k which depends on the exact speed of the (ordinary) computer being used.

(*d*) If P is a class of problems, and every instance of P can be converted (efficiently) into an instance of an NP-complete problem Q, and a solution to the corresponding instance of Q lets you (again efficiently) derive a solution to the original instance of P, then P is NP-complete.

(*e*) If P = NP then we can solve the decision version of the Travelling Salesman Problem efficiently on a deterministic computer: i.e. given a graph with weighted edges and an integer k we can find a route visiting each vertex of the graph and having total edge-weight at most k. Because of this we could then solve the minimisation version of the same problem, i.e. find the shortest path through the graph that visits each vertex, and this would still be achievable in polynomial time.

[20 marks]