

1998 Paper 6 Question 11

Complexity Theory

Here are some informally expressed opinions about computational complexity. They may be correct, incorrect, misleading or meaningless. In some cases the truth or otherwise of the statement might not be known, either in the sense of it not having been covered in the course or by the answer not being known by anybody anywhere. For each statement comment on its validity and in cases where that is both necessary and straightforward produce an adjusted version of the observation that is properly valid. You are not expected to include proofs to support your claims.

- (a) Problems that are not NP-complete are easy to solve. [3 marks]
- (b) Problems that are NP-complete will never be solved in reasonable amounts of time even though computers continue to get faster and faster. [3 marks]
- (c) To test a number N to see whether it is prime you just have to do a test-division by each of the numbers from 2 to $N - 1$, and since there are only $N - 2$ of these and division can be done in time $O(n^2)$ this is polynomial time. Thus primality testing is in the class P. [4 marks]
- (d) There is a polynomial-time reduction from the k -clique problem to 3-SAT. [3 marks]
- (e) There is a polynomial-time reduction from 3-SAT to the k -clique problem. [3 marks]
- (f) There have been proposals that biological computers based on DNA might use the massive parallelism of their biochemical activity to solve NP problems rapidly. If such systems could be made to work reliably this would solve the theoretical challenge posed by the concept of NP-completeness. [4 marks]