1998 Paper 4 Question 5

Data Structures and Algorithms

Describe and justify Dijkstra's algorithm for finding the shortest path between two vertices in a directed graph with non-negative lengths associated with its edges. [8 marks]

For the case where the nodes represent towns and the costs C_{uv} represent distances by road, Hart, Nilsson & Raphael proposed a variation where the next node to be considered is based on minimising

$$D(a) + H(a, \text{destination})$$

instead of the usual $D(a)$. $H(u, v)$ is a heuristic function which here should be taken as some constant (k, say) multiplied by the Euclidean distance between towns u and v.

Explain what benefits such a modification might bring and investigate how the correctness and speed of the modified algorithm changes with the value of k.

Can such a variation help in finding the shortest routes to all nodes from a given starting node? Justify your answer. [12 marks]