1998 Paper 11 Question 8

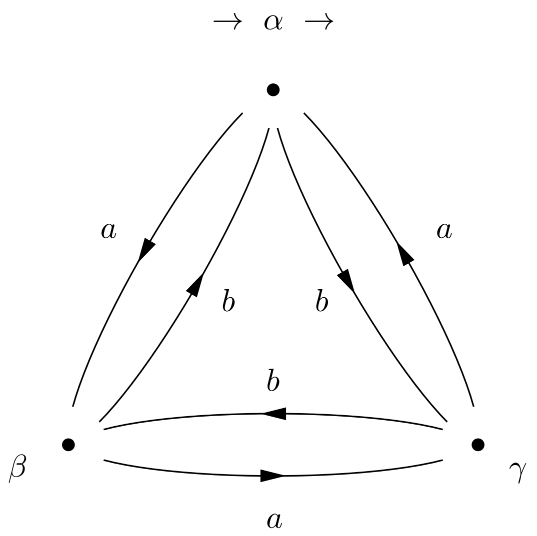
Mathematics for Computation Theory

Let $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ be an $(m+n) \times (m+n)$ event matrix partitioned so that A, D are square $(m \times m), (n \times n)$ matrices respectively. Let

$$E = (A + BD^*C)^* \qquad F = A^*B(D + CA^*B)^*$$
$$G = D^*C(A + BD^*C)^* \qquad H = (D + CA^*B)^*.$$

Show that $X = \begin{pmatrix} E & F \\ G & H \end{pmatrix}$ satisfies the event matrix equation $X = I + MX$, where I is the identity $(m+n) \times (m+n)$ matrix. [7 marks]

Consider the following deterministic finite automaton:



Here α is the initial state, and the sole accepting state. Show that the event recognised by the automaton may be described by the regular expression

$$\left\{a(ab)^*b + a^2(ba)^*a + b^2(ab)^*b + b(ba)^*a\right\}^*$$

Explain how each of the summands in the brackets arises. [13 marks]

[You may assume that if $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ is an event transition matrix partitioned so that A and D are square, then $M^* = \begin{pmatrix} E & F \\ G & H \end{pmatrix}$ takes the form stated above.]