1998 Paper 10 Question 12

Numerical Analysis I

Define *relative error*, *machine epsilon* (*macheps*). [2 marks]

Consider IEEE single-precision arithmetic. How are the 32 bits arranged in terms of sign, exponent and significand? How is the exponent stored? Explain the terms *normalized number*, *denormal number*. What is the *hidden bit* and how is it used? How are negative numbers stored? What does *NaN* stand for? Give an example of an operation that yields a *NaN* value. [6 marks]

Given that $e_{\max} = 127$, show the bit pattern representing each of the following numbers. [Draw lines to separate the sign, exponent and significand. You may use "$0 \ldots 0$" to represent long strings of zeros.]

0
$-\infty$
-1.0
$1.0 + \textit{macheps}$
4.0
$4.0 + \textit{macheps}$
1.125×2^{-31}
a *NaN* value [give one example]
$\hat{x}$, the smallest representable number greater than 2^{16}

[9 marks]

In the last case, what is

(*a*) the value of the least significant bit in the significand of $\hat{x}$, and

(*b*) the relative error if rounding error causes 2^{16} to be stored as $\hat{x}$? [3 marks]