COMPUTER SCIENCE TRIPOS Part IA

Tuesday 2 June 1998 1.30 to 4.30

Paper 2

Answer the question in Section A, **one** *question from each of Sections B and C, and* **two** *questions from Section D.*
Submit the answers in five **separate** *bundles, each with its own cover sheet. On each cover sheet, write the numbers of* **all** *attempted questions, and circle the number of the question attached.*
Write on **one** *side of the paper only.*

SECTION A

1 Twenty-part question (*One mark per part*)

(*a*) Using O-notation, specify the complexity class described by the recurrence

$$T(1) = 1$$
$$T(n) = T(n/2) + 1.$$

(*b*) How (in outline) does mergesort work?

(*c*) What are curried functions and what are their advantages?

(*d*) Use Euclid's algorithm to find the highest common factor of 221 and 247, and to express it as a linear combination of these numbers.

(*e*) Suppose that A and B are sets whose numbers of elements are a and b respectively. How many subsets does A have? How many relations are there between A and B? How many total functions are there from A to B?

(*f*) Give explicit injections from $\mathbb{N}$ to $\mathbb{N} \times \mathbb{N}$ and from $\mathbb{N} \times \mathbb{N}$ to $\mathbb{N}$.

(*g*) List the eight Java primitive types.

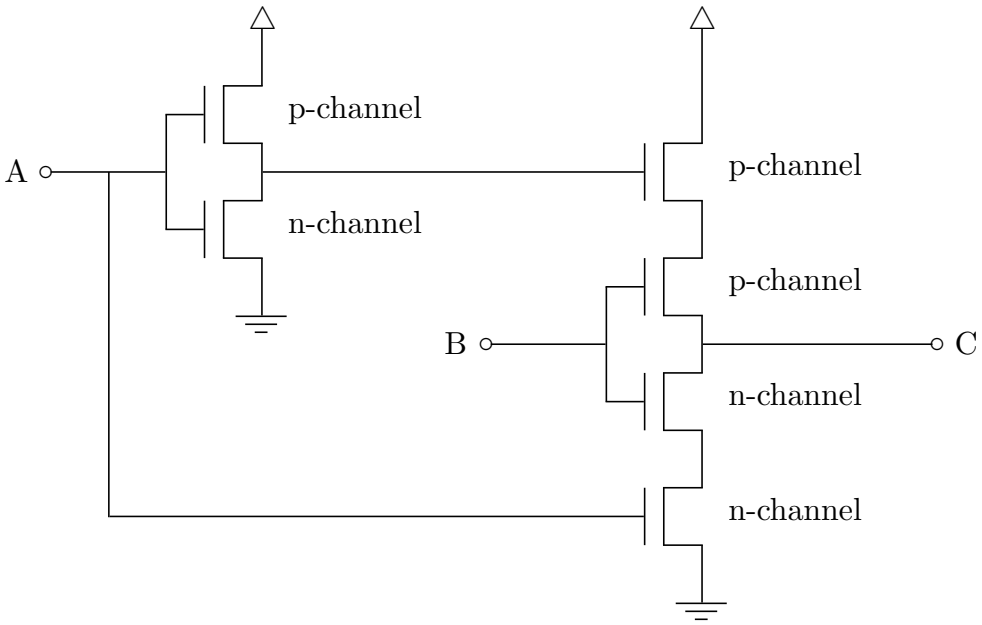
(*h*) What result will be printed if the following fragment of Java code is executed? Why?

```
double d = 6.6;
try
{   d = 1.0 / 0.0;
}
finally
{   System.out.println("d = " + d);
}
```

(*i*) What is meant by the terms *big endian* and *little endian*? Illustrate your answer by showing how the characters of the word "fleamarkets" would be represented in a machine with a 4-octet word and processor endian of each type.

(*j*) How does an I/O device which supports DMA operate?

(*k*) What is this?



(*l*) What function is represented by the following map (give the simplest form)?

	a			
b	1	1	1	1
		1	1	
		c		

(m) Why is it important for the maintenance of their professional status, that computer professionals continue to upgrade their professional knowledge and skill?

(n) Why should you never expect the following response from an ML program?

```
val it = 3.14 : integer
```

(o) If X is distributed Geometric(p), what is E(X)?

(p) If X and Y are two random variables, what is the covariance W(X,Y)?

(q) Is it more economic to have one person testing a program for 6 months, or six people testing it in parallel for 1 month?

(r) What is stepwise refinement?

(s) Give a finite deterministic automaton with alphabet of input symbols $\{a, b\}$ which accepts the language consisting of just the null string ε and the letter a.

(t) Give a reliable circuit for gating the clock to a D-type flip-flop that has the effect of a clock enable.

SECTION B

2 Digital Electronics

You are to design a 2-bit multiplier which takes inputs $b_1 b_0$ $a_1 a_0$ representing two unsigned 2-bit numbers and produces a 4-bit result in the outputs $z_3 z_2 z_1 z_0$.

(a) Give a truth table for the outputs. [5 marks]

(b) Give simplified forms for $z_3 z_2 z_1 z_0$. [5 marks]

(c) Discuss alternatives for producing an 8-bit multiplier with special consideration for gate count and speed. Give a full design for one of the alternatives for a 4-bit adder. [10 marks]

3 Digital Electronics

You are to design a circuit with the following inputs:

- D – the data input
- R – a reset input (active high)
- CLK – a clock

and one output, ERR.

ERR should be high if an error is found. An error occurs if, since the last reset, in the sequence of D values which occur on the rising edge of the clock the number of zeros exceeds the number of ones by three, or *vice versa*. R should be sampled on the rising edge of the clock. When R is asserted, the D value is ignored.

(a) Give a state diagram for the circuit. [10 marks]

(b) Implement the circuit using J-K flip flops. [10 marks]

SECTION C

4 Probability

A practical class which is conducted in Cockcroft 4 makes use of 10 DECstations and 5 PWFs. It is known that the probability of any particular DECstation failing during the class is a and the probability of any particular PWF failing during the class is b. All failures may be assumed to be independent.

After the most recent class the demonstrator reported that *two* workstations had failed. Write expressions for:

(a) the probability that both failures were of DECstations [5 marks]

(b) the probability that one DECstation failed and one PWF failed [5 marks]

(c) the probability that both failures were of PWFs [5 marks]

Hence or otherwise show that if $a = b$ the probabilities in the three cases are respectively 9/21, 10/21 and 2/21. [5 marks]

5 Probability

The Gambler's Ruin Problem presupposes a game for two players, A and B, each of whom has a pile of £1 coins. The game proceeds by a sequence of turns and at each turn A wins with probability p and B wins with probability q (and $p+q=1$). When the outcome of a turn is known, £1 is transferred from the loser's pile to the winner's pile. Play continues until one player is ruined by having no money left.

At a particular stage in the game A has £n and B has £$(a-n)$. If u_n is the probability that A ultimately wins from this position the following difference equation holds:

$$u_n = p\,u_{n+1} + q\,u_{n-1} \qquad \text{provided } 0 < n < a$$

The right-hand side makes use of the multiplication theorem twice and the addition rule once. Explain why such usages are justified. [3 marks]

Provide suitable boundary conditions to reflect the two possible end-of-game outcomes. [2 marks]

Suppose that d_n represents the duration of play, the expected number of turns from the position described until the end of the game. Write down the related difference equation for d_n and provide suitable boundary conditions. [4 marks]

Solve the revised difference equation for the fair case where $p = q = \frac{1}{2}$. [8 marks]

Ten children sit in a circle to play pass the parcel. At each turn, the child who holds the parcel passes it one child to the left or right with equal probability. At the start of the game, at turn zero, a particular child (the leader) is handed the parcel by an outsider. At what turn number may the leader expect to receive the parcel again? The term "expect" is used in the probabilistic sense of expectation. [3 marks]

SECTION D

6 Professional Practice and Ethics

What are the moral and practical (or legal) difficulties in the private ownership of software? Are there any alternatives to private ownership of software? [20 marks]

7 Regular Languages and Finite Automata

Explain how the Pumping Lemma is used in proofs that languages are not regular. [3 marks]

State, with justification, whether each of the following statements is true or false.

(*a*) $\{a^m b^{2n} \mid m \geqslant 0 \text{ and } n \geqslant 0\}$ is regular.

(*b*) $\{a^p b^{2q} \mid p, q \text{ prime}\}$ is regular.

(*c*) No infinite subset of $\{a^n b^n \mid n \geqslant 0\}$ is regular.

(*d*) No infinite subset of $\{ww \mid w \in \{a, b\}^*\}$ is regular.

(*e*) Every finite subset of $\{ww \mid w \in \{a, b\}^*\}$ is regular.

[17 marks]

8 Software Engineering I

Discuss the lessons learned from the London Ambulance Service disaster under the following headings:

(*a*) capturing user requirements

(*b*) project management

(*c*) quality assurance

(*d*) testing [16 marks]

What in your view would be the single most important measure to take in developing a mission-critical system in order to reduce the likelihood of such a disaster? [4 marks]

9 Software Engineering II

Consider the following code fragment, whose purpose is to form the product $m \times n$ in the integer variable x:

```
x := 0;  i := n;
while i<>0 do
    begin
    x := x+m;
    i := i-1
    end;
```

What is the loop invariant? [3 marks]

Argue informally that the invariant holds at the appropriate points. Using the invariant, show that the loop is correct. [5 marks]

Outline the main features of the Z specification language. [9 marks]

To what extent can a formal specification language help a safety-critical system meet the requirement of being extremely reliable? [3 marks]

10 Structured Hardware Design

The user interface to a microwave oven consists of a washable membrane keyboard and an LCD panel. All of the interface electronics is to be integrated onto a single ASIC (application-specific integrated circuit).

(a) Briefly sketch or describe the overall structure of the microwave oven and define the connections to the ASIC. [5 marks]

(b) The keyboard has 25 keys that are simple push-to-make switches. For minimal pin use, these are to be scan multiplexed. Sketch or describe the way the keypad is connected to the ASIC and the circuitry to scan the keypad. How is debouncing handled? [12 marks]

(c) If the main functions within the ASIC are implemented with software on an imbedded microprocessor, discuss whether it is worthwhile having dedicated electronics to scan the keyboard and display. [3 marks]