1997 Paper 8 Question 3

Computer System Modelling

A group of smart computer scientists decide to earn some money in the vacation by buying a punt and running a chauffeured punt tour operation on the Cam. Based at Magdalene Bridge, they run tours upriver showing tourists the delights of the Backs. As those familiar with punt trips will know, journeys take an exponentially distributed time to complete, particularly when accompanied by strawberries and champagne. In this case our punt operators have measured that the average trip takes 30 minutes. Tourists (being tourists) behave rather randomly and arrive at the quay independently, according to Poisson distribution with mean 10 per hour. Each punt can accommodate six tourists, but departs as soon as there is at least one tourist wanting to take the tour.

(a) Draw a Markov chain model for the punt tour operation, annotating it with the appropriate rates and state information. Briefly explain the diagram.

[5 marks]

(b) Propose a model which can safely be used as a worst-case approximation to the system above. Why is your model conservative? Using your new model, calculate the expected length of the queue of tourists. What constraints are required to ensure that the queuing system remains stable? [5 marks]

(c) Now assume that tourists do not join the queue if there are already six others waiting. If each tourist pays £5 per trip, and a punt costs £10 per hour to run, calculate the expected hourly profit of the punt company. Propose a scheme which would enable the entrepreneurs to increase their profits, but do not solve a model of the new system. What is the utilisation of the punts in the old and new schemes? [10 marks]