1997 Paper 6 Question 12

Semantics of Programming Languages

A language L of expressions, E, for infinite lists of integers is given by:

$$E ::= \mathbf{ints} \mid \mathbf{from}(n) \mid n : E \mid \mathbf{incr}(E)$$

where n ranges over the integers. A binary relation of evaluation between L-expressions is inductively defined by the following axioms and rule:

$(\Downarrow_{\text{ints}})$ $\quad \mathbf{ints} \Downarrow 0 : \mathbf{incr}(\mathbf{ints})$.

$(\Downarrow_{\text{from}})$ $\quad \mathbf{from}(n) \Downarrow n : \mathbf{from}(n') \quad$ where $n' = n + 1$.

$(\Downarrow_{:})$ $\quad n : E \Downarrow n : E$.

$(\Downarrow_{\text{incr}})$ $\quad \dfrac{E \Downarrow n : E'}{\mathbf{incr}(E) \Downarrow n' : \mathbf{incr}(E')} \quad$ where $n' = n + 1$.

Prove that for every E, there are unique n and E' such that $E \Downarrow n : E'$. [5 marks]

A binary relation $\mathcal{R}$ between L-expressions is called a *bisimulation* if whenever $(E_1, E_2) \in \mathcal{R}$ then $E_1 \Downarrow n : E_1'$ and $E_2 \Downarrow n : E_2'$ hold for some n and some $(E_1', E_2') \in \mathcal{R}$. We write $E_1 \approx E_2$ if E_1 and E_2 are related by some bisimulation.

Prove that $\mathbf{ints} \approx \mathbf{from}(0)$. [5 marks]

Prove that $\approx$ has the congruence property for the language L, i.e. that if $E_1 \approx E_2$, then $E[E_1] \approx E[E_2]$ (where $E[E_1]$ is any L-expression containing an occurrence of E_1 and $E[E_2]$ denotes the result of replacing that occurrence by E_2). [Hint: show that $\approx$ is preserved by the operations $n : -$ and $\mathbf{incr}(-)$, by constructing suitable bisimulations.] [5 marks]

State, with justification, whether or not $\approx$ is an equivalence relation. [5 marks]