1997 Paper 5 Question 11

Foundations of Functional Programming

(*a*) Give the definition of a head-normal form and head reduction of a λ-term. Argue that every normal form is a head-normal form. [4 marks]

(*b*) Let $Y_M \equiv \lambda f.WWM$, where $W \equiv \lambda x.\lambda z.f(xxz)$ and M is an arbitrary term. Give a head-normal form and normal form for the following λ-terms, or indicate why they do not exist:

(*i*) Y_M

(*ii*) $Y_M(KI)$, where $K \equiv \lambda x.\lambda y.x$ and $I \equiv \lambda x.x$

(*iii*) $Y_M(K)$, where K is as above

[You may assume that head reduction always terminates when a head-normal form exists.] [6 marks]

(*c*) A λ-term is solvable if there exist variables $x_1, \ldots, x_n$ and λ-terms $N_1, \ldots, N_m$ for $n, m \geqslant 0$ such that $(\lambda x_1 \ldots \lambda x_n.M)N_1 \ldots N_m = I$.

Show that every head-normal form is solvable. [4 marks]

For each term in (*b*), prove that it is solvable or that it is unsolvable. [6 marks]