1997 Paper 4 Question 8

Computation Theory

Define what is meant by saying that a set of *partial recursive* (μR) functions is *recursively enumerable*. Explain briefly how the universal register machine might be used to define a universal μR function $\mu(e, x)$ that enumerates the set of *all* partial recursive functions of a single variable x. [6 marks]

(*a*) Prove that the set of all total recursive functions of a single variable is not recursively enumerable. [4 marks]

(*b*) Show that there are recursively enumerable sets that are not recursive. [6 marks]

(*c*) Show that there is a partial recursive function that cannot be extended to any total recursive function. [4 marks]

[Any properties of recursively enumerable sets that you assume should be clearly stated.]