1997 Paper 2 Question 5

Probability

Suppose $f(x)$ is a probability density function associated with a continuous random variable X and $y(x)$ is a transformation function whose inverse is the function $x(y)$. The derived random variable $Y = y(X)$ will be associated with some probability density function $g(y)$ where:

$$g(y) = f\big(x(y)\big)\left|\frac{dx}{dy}\right|$$

Suppose that X is distributed Uniform(0,1) and, accordingly, has an associated probability density function $f(x)$ given by:

$$f(x) = \begin{cases} 1, & \text{if } 0 \leqslant x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Suppose, further, that Y is required to be distributed as a triangular distribution such that the probability density function $g(y)$ is:

$$g(y) = \begin{cases} 1 + y, & \text{if } -1 \leqslant y < 0 \\ 1 - y, & \text{if } 0 \leqslant y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Determine the transformation function $y(x)$ that satisfies this requirement.

[15 marks]

An experienced gambler notes that when two fair and independent dice are thrown the sum S of the two scores is distributed as a similar, albeit discrete, triangular distribution. S can be transformed into a derived random variable $T = \alpha S + \beta$ (where α and β are constants).

Take the range of values of S as 1 to 13 and observe that the probability of each of these outcomes is zero. What values of α and β ensure that the discrete distribution of T corresponds most closely to the continuous distribution of Y? [5 marks]