1997 Paper 11 Question 10

Numerical Analysis I

Explain the terms *unit round off* and *machine epsilon* (*macheps*). Why is *machine epsilon* used in preference to *unit round off* for practical purposes?   [4 marks]

In the IEEE binary floating-point Standard (*IEEE* 754), what *exponent* and *significand* are used in representing each of the numbers 0, 1 and 2 in single precision? How are the exponent and significand stored in each case?   [6 marks]

Show the 32 bits that represent (1+*macheps*). What is the *exact* value of *macheps* in this case?   [4 marks]

What are the two sources of error in the formula

$$f'(x) \simeq \frac{f(x+h) - f(x)}{h}$$

and how does each type of error behave as h increases?   [4 marks]

Suggest a suitable value of h if using this formula with IEEE single precision when $f(x) = O(1)$.   [2 marks]