

# COMPUTER SCIENCE TRIPOS Part II

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Thursday 5 June 1997 1.30 to 4.30

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Paper 9

*Answer **five** questions.*

*Submit the answers in five **separate** bundles, each with its own cover sheet. On each cover sheet, write the numbers of **all** attempted questions, and circle the number of the question attached.*

*Write on **one** side of the paper only.*

## 1 Distributed Systems

Discuss the tradeoffs that can be made between *consistency* and *availability* of replicated data in a distributed system. Your discussion should cover (a) the requirements of different applications and (b) the algorithms and protocols that can be used to implement the required policies. [20 marks]

## 2 Digital Communication II

To what extent can a remote procedure call (RPC) be made to look like a local procedure call? [5 marks]

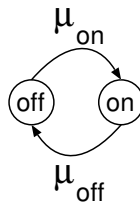
Consider an RPC system running over an unreliable transport service. What are the communication errors that can arise and how can they be handled? [5 marks]

What other system failures can arise and how can they be handled? [5 marks]

Describe the steps a programmer uses to take a local procedure call and turn it into a remote procedure call, using appropriate tools. [5 marks]

### 3 Computer System Modelling

The two-state Markovian on-off source is used in computer networking research as a model of bursty network traffic. The source is pictured in the figure below. When in the on state it generates fixed-sized network packets at a *constant* rate of 100 packets/second. Each set of packets generated in the on state is called a *burst*. When in the off state the source is silent. The residence time in each state is exponentially distributed. The *burstiness* of the source is defined as the ratio of its peak and mean rates.



A traffic modeller wants to write a simulation module which emulates this source, with the requirement that it transmits bursts of traffic with a mean length of 25 packets, and a burstiness of 20.

- (a) Calculate the rates  $\mu_{\text{off}}$  and  $\mu_{\text{on}}$ . [10 marks]
- (b) Briefly describe how to generate values which are distributed exponentially for use in the simulator. [3 marks]
- (c) State Little's Law. Outline a proof of Little's Law with the aid of a diagram. [7 marks]

### 4 VLSI

Write short notes on the advantages and disadvantages of *each* of the following approaches to clocking a VLSI chip:

- (a) single global clock
- (b) two-phase clock
- (c) synchronised clock using an on-chip phase-locked loop
- (d) self-timed logic (no clock)

[5 marks each]

## 5 Business Studies

Describe the rôles and relationships of the members of a Chief Programmer team. [5 marks]

Describe Maslow's *Hierarchy of Needs*. [5 marks]

Discuss stages of group formation. How does belonging to a group satisfy some of the needs described by Maslow? [5 marks]

"The problems of co-ordination mean that there is a limit on the size of any software system that can be successfully built." Discuss. [5 marks]

## 6 Advanced Algorithms

Describe the structure of a *Fibonacci heap* and explain how to perform the following operations:

- (a) create an empty heap
- (b) add a single new node to a heap
- (c) form the union of two existing heaps
- (d) identify the smallest item in a heap

How long does each of these operations take?

[20 marks]

## 7 Optimising Compilers

Explain what is meant by an *effect system* for a typed language. Distinguish between immediate effects and possible other effects; also give a typical form of sequent  $\Gamma \vdash e : \langle \textit{whatever} \rangle$ . [4 marks]

Given the following subset of ML,

$$e ::= x \mid \lambda x.e \mid e \ e' \mid \textit{let } x = e \textit{ in } e' \mid \textit{if } e \textit{ then } e' \textit{ else } e'' \mid \textit{ref } e \mid !e \mid e := e',$$

design an effect system for terms  $e$ , for which the (immediate) effects of an expression are any subset of  $\{C, R, W\}$  representing reference creation, dereferencing and assignment to some reference cell. You may assume that the ML-like types  $t$  of the language involve integers, functions and reference types but have no polymorphism. Assume also that assignment returns the value assigned. It suffices to give clauses for  $x$ ,  $\lambda x.e$ ,  $e \ e'$ , *if*  $e$  *then*  $e'$  *else*  $e''$  and  $e := e'$ . [6 marks]

Explain how your system copes with terms like

$$\lambda x.\lambda y.\textit{if } x \textit{ then } y := 1 \textit{ else } 0$$

and

$$\lambda x.\lambda y.\textit{if } x \textit{ then } \lambda z.y := z + 1 \textit{ else } \lambda z.0.$$

(If your system cannot handle these cases then instead explain how one might adjust it to do so.) [4 marks]

Explain how the analysis might be used to determine when the optimisation of  $e + e$  to *let*  $x = e$  *in*  $x + x$  is safe. [3 marks]

Similarly, suggest a criterion on the type or effect of  $f$  in *let*  $f = \lambda x.e$  *in*  $f(1) + f(2)$  which would enable the two calls to  $f$  to be evaluated concurrently. [3 marks]

## 8 Neural Computing

Discuss how neural operators which encode, analyse and represent image structure in natural visual systems can be implemented in artificial neural networks. Include *four* of the following issues in your discussion:

- receptive field structure
- adaptiveness and perceptual learning
- hierarchies of tuning variables in successive layers of the visual pathway
- wavelet codes for extracting pattern information in highly compressed form
- self-similarity of weighting functions
- associative memory or content-addressable memory for recognising patterns such as faces and eliciting appropriate response sequences

[20 marks]

## 9 Security

Write brief notes on *each* of the following:

- (a) the Internet Worm
- (b) Trojan horses
- (c) polymorphic viruses
- (d) virus exploitation of covert channels
- (e) the distinguishing characteristics of viruses written in interpreted languages

[4 marks each]

## 10 Denotational Semantics

Define the types and terms of the language PCF. Describe the denotational semantics of PCF using domains and continuous functions. In what sense is the denotational semantics of PCF *compositional*? [12 marks]

Explain the *soundness* and *adequacy* properties of the denotational semantics with respect to the operational semantics of PCF. (A definition of the PCF operational semantics need not be given.) [4 marks]

Define the notion of *contextual equivalence* for PCF terms. Explain why the compositional, soundness and adequacy properties mentioned above imply that if two closed PCF terms of the same type have equal denotation, then they are contextually equivalent. [4 marks]

## 11 Information Theory and Coding

The information in continuous but bandlimited signals is *quantised*, in that such continuous signals can be completely represented by a finite set of discrete numbers. Explain this principle in *each* of the following four important contexts or theorems. Be as quantitative as possible:

- (a) The Nyquist Sampling Theorem. [5 marks]
- (b) Logan's Theorem. [5 marks]
- (c) Gabor Wavelet Logons and the Information Diagram. [5 marks]
- (d) The Noisy Channel Coding Theorem (relation between channel bandwidth  $W$ , noise power spectral density  $N_0$ , signal power  $P$  or signal-to-noise ratio  $P/N_0W$ , and channel capacity  $C$  in bits/second). [5 marks]

## 12 Computer Vision

Discuss the rôle of non-linear operators in vision for the extraction of motion information, texture information, colour information, and stereo information. What are the limitations of linear operators (such as filters) compared with non-linear ones? What is a quadrature pair, and what is a Hilbert pair? What is a Hilbert Transform, and what is a natural way to build a useful non-linear operator from it? [20 marks]

### 13 Types

Give the syntax of (types and terms of) the second-order polymorphic lambda calculus  $\lambda 2$  whose five ways of constructing terms,  $M$ , are: identifiers, lambda abstraction, application, type abstraction and type application. (The last two are sometimes known as *generalisation* and *specialisation*.) Make it clear which, if any, sub-phrases of terms represent types or type variables. [4 marks]

Give a term  $M$  conforming to the syntax of  $\lambda 2$  which is not well-typed according to the usual inference rules for  $\lambda 2$ . [2 marks]

Let  $\lambda U$  be the untyped lambda calculus whose terms  $N$  have syntax:

$$N ::= x \mid \lambda x.N^1 \mid N^1 N^2.$$

Define a function  $erase : \lambda 2 \rightarrow \lambda U$  which removes all types from a  $\lambda 2$  term, but which preserves the rest of it.

[Hint:  $erase(\Lambda\alpha.M) = erase(M)$ .] [3 marks]

Now find (or briefly justify why this is impossible):

- (a) two well-typed  $\lambda 2$  terms  $M_1$  and  $M_2$  without free type variables such that  $erase(M_1) = erase(M_2) = \lambda x.x$  and that  $M_1$  and  $M_2$  differ by more than type variable renaming;
- (b) a well-typed  $\lambda 2$  term  $M_3$  such that  $erase(M_3) = \lambda x.xx$ ;
- (c) a well-typed  $\lambda 2$  term  $M_4$  such that  $erase(M_4) = (\lambda x.xx)(\lambda x.xx)$ ;
- (d) a well-typed  $\lambda 2$  term  $M_5$  such that  $N_5 = erase(M_5)$  has no ML type;
- (e) a  $\lambda U$  term  $N_6$  which has an ML type, but such that there is no well-typed  $\lambda 2$  term  $M_6$  with  $erase(M_6) = N_6$ .

[11 marks]

## 14 Numerical Analysis II

Explain the term *positive semi-definite*. [1 mark]

Let  $\mathbf{A}$  be a square matrix. State *Schwarz's inequality* for the product  $\mathbf{Ax}$ . What are the *singular values* of  $\mathbf{A}$ , and how are they related to the  $\ell_2$  norm of  $\mathbf{A}$ ? [4 marks]

Describe briefly the *singular value decomposition* of the matrix  $\mathbf{A}$ , and how it may be used to solve the linear equations  $\mathbf{Ax} = \mathbf{b}$ . [4 marks]

Let  $\hat{\mathbf{x}}$  be an approximate solution of  $\mathbf{Ax} = \mathbf{b}$ , and write  $\mathbf{r} = \mathbf{b} - \mathbf{A}\hat{\mathbf{x}}$ ,  $\mathbf{e} = \mathbf{x} - \hat{\mathbf{x}}$ . Find an expression for the relative error  $\|\mathbf{e}\|/\|\mathbf{x}\|$  in terms of computable quantities. Show how your formula is related to the *singular values* of  $\mathbf{A}$ . [8 marks]

How may this formula be used if some *singular values* are very small? [3 marks]

## 15 Communicating Automata and Pi Calculus

Define the notions of *sort* and *sorting* for the  $\pi$ -calculus, and explain what is meant by the assertion that a process  $P$  *respects* a sorting. Give *two* reasons why sorting is useful. [7 marks]

Simple data values can be represented as abstractions in the  $\pi$ -calculus. In particular, if *True* and *False* are abstractions representing the two truth-values, then  $b.True$ ,  $b.False$  are processes in which each truth-value is located at  $b$ .

Define the abstractions *True* and *False*. Also, for arbitrary processes  $P$  and  $Q$ , define the abstraction  $CASES(P, Q)$  such that

$$\begin{aligned} CASES(P, Q)\langle b \rangle \mid b.True &\longrightarrow^* P \\ CASES(P, Q)\langle b \rangle \mid b.False &\longrightarrow^* Q \end{aligned}$$

and demonstrate these reductions. Give a sorting respected by all these constructions. [6 marks]

Discuss, with technical details, the general method by which  $\pi$ -calculus abstractions may also be used to represent compound data structures such as lists. [7 marks]